An item analysis of the competition problems was completed at the state tournament. The responses analyzed included the 45 multiple-choice problems on the written test and the 10 problems from the individual ciphering round. Before we discuss what the item analysis revealed, some background information would be useful. The problems on the written test are designed to increase in difficulty. Thus, theoretically, problem 1 is the easiest multiple-choice problem and problem 45 the most difficult multiple-choice problem on the test. Below are those problems.

Test Problem #1: In Mathematicity, the capital of Mathland, on the famous boardwalk Hilbert Street, there are six exclusive hotels: the Hotel Abel, the Brahmagupta Ballroom, the Cauchy Grand Hotel, the Descartes Inn, Euler Suites, and Fermat Towers. Each hotel has 12 rooms available for booking for the Math Contest Problem Writers Convention. Compute the minimum number of rooms needed to be booked to guarantee that Hotel Abel gets at least 2 rooms reserved.

a) 2 b) 7 c) 14 d) 62 e) 74

According to the analysis, problem 1 was not the easiest, as 142 students out of 157 answered it correctly. Problem 7 narrowly earns that distinction: 145 out of the 157 participants answered the question correctly.

Test Problem #7: Determine the domain of the function $f(x) = 2/(x^{1/2} - (4 - x)^{1/2})$.

a) all real numbers b) $[1, 3]$ c) $(0, 4)$ d) $[0, 4]$ e) $[0, 2) \cup (2, 4]$

In contrast, the analysis revealed that Problem 45 really was the most difficult since only 9 participants answered it correctly.

Test Problem #45: The infinite sum $\sum \arctan(2/(n + 1)^2)$, where $n = 1$ to infinity, can be written in the form $p\pi/q$ where p and q are integers and p/q is a reduced fraction. Compute $p + q$.

a) 4 b) 5 c) 6 d) 7 e) 8

Problem 1 is a straightforward application of the "pigeonhole principle". The worst case scenario for the Hotel Abel is that all the rooms at all the other hotels fill up first, leaving only 2 rooms for the Hotel. Thus, it takes 62 rooms for the Hotel to get two of them and the answer is D. Problem 7 requires recognizing that not only the expression within each square root cannot be negative, but the entire denominator cannot be zero. Hence, the answer is E. Problem 45 requires clever manipulation of $2/(n + 1)^2$ so that one may use the tangent sum of angles identity and thus convert the series into one which telescopes. The correct answer is D.

As for the ciphering, there is no particular order of difficulty for the questions, so it is always interesting to see which problems are answered correctly and quickly. The easiest ciphering problem, judged by the fact that 112 participants gave the correct answer, is the following. (Recall that each of the problems below should be answered in less than two minutes, without a calculator.)

Ciphering Problem #1: Solve $x! = (5!)! / 5!$.

Since $5! = 120$, we have $x! = 120! / 120 = 119!$, so that $x = 119$. The most difficult ciphering problem, with only 10 students giving the correct answer of 3969π, was the following.

Ciphering Problem #7: A regular hexadecagon (16-sided polygon) with a perimeter of 2016 is inscribed by a circle of radius r and is circumscribed by a circle of radius R. Compute the positive difference of the areas of the two circles.

The solution to this problem involves recognizing that one must split the hexadecagon in to 16 congruent isosceles triangles with base 126, altitude r and congruent sides of length R.