

Item Analysis

The responses analyzed included the 45 multiple-choice problems on the written test and the 10 problems from the individual ciphering round. Before we discuss what the item analysis revealed, some background information would be useful. The problems on the written test are designed to increase in difficulty. Thus, theoretically, problem 1 is the easiest multiple-choice problem and problem 45 the most difficult multiple-choice problem on the test. Below are those problems.

Test Problem #1: Given that $x^2 + 4x + 18 = 38$, compute the value of $5x^2 + 20x + 8$.

- a) 38 b) 76 c) 108 d) 190 e) 400

Test Problem #45: Suppose I have two independent Gaussian random variables X and Y with means of 50 and 90, respectively, and the same variance σ^2 . Suppose Z is a bimodal Gaussian random variable where if I flip a coin and get heads, I sample from X and if I get tails, I sample from Y . Which of the following is closest to the value of σ^2 which gives $P_z(Z = 50) = P_z(Z = 70)$ and $P_z(Z = 70) = P_z(Z = 90)$, where $P_z(Z)$ is the probability density function of the random variable Z ?

- a) 20 b) $200 / \ln 5$ c) $200 / \ln 3$ d) 200 e) $200 / \ln 2$

According to the analysis, problem 1 was actually the easiest. However, even though there were only 3 correct responses to problem 45, it was not actually the most difficult. Instead, problem 43 was the most difficult, which is stated below.

Test Problem #43 (the most difficult): Every morning, Steve walks 1 mile from his house to school. If it is raining and he has an umbrella at his house, he brings it with him, otherwise he gets rained on. Every evening, Steve walks 1 mile from school back to his house. If it is raining and he has an umbrella at school, he brings it with him, otherwise he gets rained on. Currently, Steve has one umbrella at his house and one umbrella at school. Each morning or evening, the probability of it raining is $\frac{1}{2}$. What is the expected number of miles Steve walks before getting rained on for the first time?

- a) 3 b) 5 c) 7 d) 9 e) 11

Problem 1 is a straightforward algebraic manipulation problem. One only has to multiply the given equation by 5, and then subtract 82 from both sides; it is not necessary to find the value of x . Of the 154 participants, 149 responded with the correct answer, C. Problem 43 requires knowledge of probability and a clever use of expected value. Only 2 students responded with the correct answer, E.

As for the ciphering, there is no particular order of difficulty for the questions, so it is always interesting to see which problems are answered correctly and quickly. The easiest ciphering problem, judged by the fact that 91 participants gave the correct answer, is the following. (Recall that each of the problems below should be answered in less than two minutes, without a calculator.)

Ciphering Problem #10: Given that w , x , y , and z are distinct prime numbers where $zy + x = w$, compute the smallest possible value of $w + x + y + z$.

Since the smallest sum must involve the primes which make the power the smallest, we must have $z = 2$ and $y = 3$; then $x = 5$ and $w = 13$. The sum is therefore 23. The most difficult ciphering problem, with only 8 students giving the correct answer of 70, was the following.

Ciphering Problem #6: Find the minimum value of $25\sin(x) + 16/\sin(x) + 25\cos(x) + 9/\cos(x)$ for $0 < x < \pi/2$.

The solution to this problem involves recognizing that one must cleverly employ the arithmetic-geometric mean inequality.