Estimating Square Roots with Square Tiles

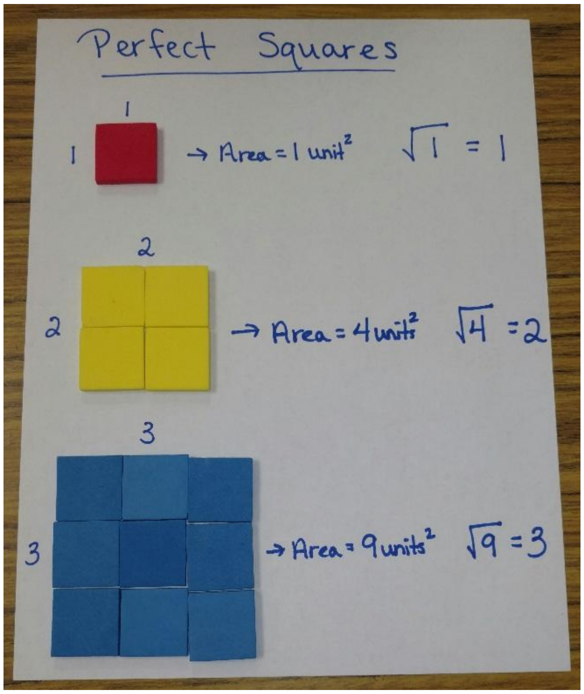
By Ashley Clody

The development of students' understanding as the Georgia curriculum has become more conceptually based over the past 10 years. Students need to do more physically engaging math rather than just sitting and listening to a lesson every day. They are movers and doers which is why the tools and manipulatives are so important to their conceptual understanding of the math content.

As a middle school teacher, I am always looking for tools and strategies that help my students understand mathematics conceptually. I feel that if I am always striving to learn something new, it motivates my students to learn new things. There are tools and manipulatives that I have used for certain activities, but I am beginning to find new ways to use them. I have used square tiles with 6th grade to show perimeter and area, but I never imagined they could be used to help 8th grade students conceptually understand estimating square roots. I will say I cannot take full credit for the idea; I learned this strategy from a fellow colleague. I have found some teachers are unaware of this strategy, and so I wanted to share it with the GCTM community to help others try and use square tiles with their students. In particular, one meaningful benefit of these manipulatives is their application to teaching students about irrational roots on the number line.

This lesson addresses the state standard MGSE8.NS.2. I like using the square tiles because of the different colors. I have my students use three different colored tiles throughout this activity.

I opened the lesson with a review of the perfect square numbers and ask students to create the first three perfect squares with the tiles. We discussed how each tile was one unit by one unit and had an area of one unit squared. Students then used their tiles to create a two by two and three by three square. I also asked them what the areas of those other two squares were. By this point, they were visually seeing the relationship between our square areas and our perfect square roots.

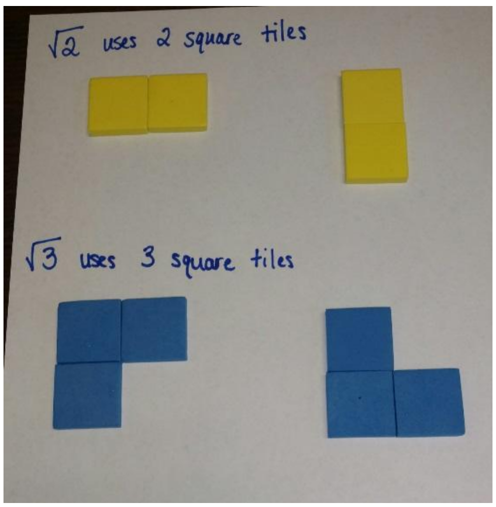


As the main lesson, we worked through creating models for irrational roots. I started by asking the students about those irrational roots.

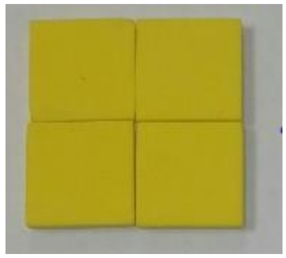
I ask questions like:

- What two integers is the $\sqrt{2}$ between?
- What two integers is the $\sqrt{6}$ between?
- Is the $\sqrt{12}$ closer to 3 or 4?

Once students became familiar identifying the integer values these irrational roots fell between, I proceeded to have them create models for them. I began by demonstrating that the irrational numbers could not create a perfect square with equal whole number dimensions. As you can see in the picture below, there is no way to take two square tiles or three square tiles and make a perfect square with whole numbers as lengths.

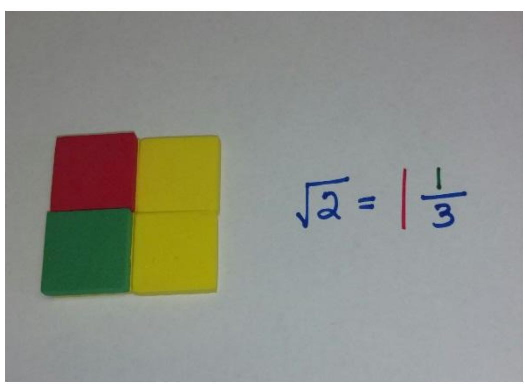


Since we know that both $\sqrt{2}$ and $\sqrt{3}$ are bigger than one and less than two, I explained to the class that these two irrational roots would have a fractional part to them. We know that the fraction will be bigger than one and smaller than two, so I had them recreate $\sqrt{4}$ with their square tiles.

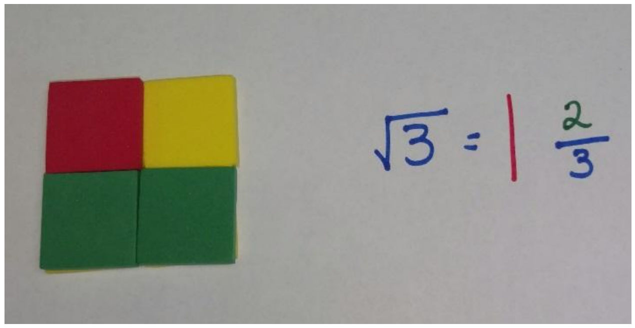


I have students visually see that $\sqrt{1}$ takes up a part of $\sqrt{4}$.

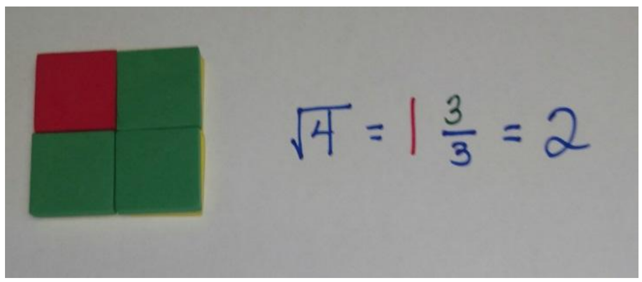
The leftover yellow squares of $\sqrt{4}$ are the parts of the remaining whole of two. By counting the number of yellow tiles, we can see that our remaining whole is made of three parts. If I want to create $\sqrt{2}$ we must fill in one part of the remaining whole. So this has a fractional representation of $1\frac{1}{3}$ because we used $\sqrt{1}$ which equals one and then one of the remaining three yellow tiles to create the fraction $\frac{1}{3}$. Students can then determine that an estimate for $\sqrt{2}$ is about 1.3 as a decimal.



As a class we continue the process for $\sqrt{3}$ and determine that it has a fractional representation of $1\frac{2}{3}$ or about 1.7 as a decimal.



After noticing a pattern starting, I had the students demonstrate $\sqrt{4}$ with the green tiles to show how we use all fractional parts of the yellow tiles to create the second whole.



I continued the lesson by asking the following questions:

- If we continue to find the other irrational roots, will the same pattern occur?
- Will our denominator always be 3?
- What will definitely change as we find irrational roots like $\sqrt{5}$, $\sqrt{6}$, and so on?

I wanted the class to see that our denominator will not always be three and we will see a similar pattern because our denominator is formed by the differences of the two perfect roots. I also wanted them to see that our whole number will definitely change as we look at larger irrational roots because they are now between two different perfect roots.

I started the next set of irrational roots with them with the three by three square as the whole and the two by two square taking up a portion. Students could see a pattern in that the denominator of the fraction ended up being the difference of the perfect root value. For example, with $\sqrt{4}$ and $\sqrt{9}$, there is a difference of 5. That is the number of fractional parts to the next remaining whole. Students also began to see that each new irrational root just added on an additional part of the whole.

I heard great conversations happening I walked around to their groups like, “Look, the extra tiles is the difference of four and nine.” I even heard some groups say, “The numerator is just one more than the previous root.”

I had all students create estimated values for all roots up to $\sqrt{25}$. For those students who picked up on the patterns faster than others, I gave them some more challenging irrational roots where they didn’t have enough tiles to visually create them. I told them to think about what they discovered with the tiles and the patterns they saw, and most groups were able to find the fractional representations of the higher roots.

For example, I asked them to find $\sqrt{104}$ by thinking that it is between ten and eleven. They noticed that it was four more than $\sqrt{100}$ or ten, and that the difference between $\sqrt{100}$ and $\sqrt{121}$ was 21. So they determined the estimate to be $10\frac{4}{21}$.

I closed the lesson by reviewing the patterns we noticed and giving a few other roots for the group to find as a whole. I also related this process and those patterns to how they could also use the number line to also see the same relationship.

For example, if we wanted to find $\sqrt{14}$ on the number line, we would determine what integers it is between. In this case, it would be between three and four. $\sqrt{9}$ is equal to three and $\sqrt{16}$ is equal to 4. I had students add these values above three and four on the number line. The difference between 9 and 16 is 7 so that becomes our denominator, and 14 is 5 more than 9 to give us our numerator. Therefore, $\sqrt{14}$ will be $3\frac{5}{7}$.

I have found that this activity conceptually helps students with finding an estimate to square roots. It does take them some time, and they do get a little frustrated at first. But, once they determine the patterns between the values, they realize that the models can be applied abstractly to all irrational roots and they can find any root they want.