



REFLECTIONS

REFLECTIONS

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MATHEMATICS:
CONNECTING THE PIECES

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REFLECTIONS

BETTER REFLECTIONS?

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President's Welcome

by Lynn Stallings
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What's Happening in Mathematics Education in Georgia?

Standards, Certificate Upgrades, Race to the Top, Etc.

So much is going on in Georgia right now that it's hard to keep up with it all! In this column, we'll discuss Georgia's proposed alternative Math IV courses, changes to certification requirements, and legislative news related to elementary mathematics endorsement and PLUs. Other news includes an update on the Race to the Top and the Common Core State Standard Initiative.

Georgia Performance Standards Math IV Options

Students who successfully complete Math I, II, and III will have the option of taking Math IV, AP Statistics or several other interesting alternatives. These alternative fourth-year mathematics courses will prepare students for non-mathematics college majors, technical training, or a variety of other possible career options. The Georgia Department of Education has released the following course titles and descriptions (quoted from a GDOE 2/9/10 release). School systems will have the option of deciding which of these courses to offer.

Advanced Mathematical Decision Making

This course is designed to follow the completion of Mathematics I, II, and III. It will prepare students for non-mathematics intensive college majors, for technical training, or for a range of career options. AMDM gives students 21st century tools to deal with the numerical information we see every day. It will help prepare students to make informed decisions as a voter and participant in society. This course deals with a variety of topics, such as voting and polling; understanding credit, debt, and investments; managing data; and network graphs. Developed by the Dana Center at the University of Texas, Austin, this course "includes descriptive statistics, financial/economic literacy, and basic trigonometry, with a heavy emphasis on using algebraic, geometric, and statistical models for a range of situations and problems."

Advanced Mathematical Decision Making in Industry and Government

This course is designed to follow the completion of Mathematics I, II, and III. Modeled after operations research

courses, AMDM in Industry and Government allows students to explore decision making in a variety of industries such as: Airline--scheduling planes and crews, pricing tickets, taking reservations, and planning the size of the fleet; Pharmaceutical --R& D management; Logistics companies--routing and planning; Lumber and wood products--managing forests and cutting timber; Local government--deployment of emergency services, and Policy studies and regulation--environmental pollution, air traffic safety, AIDS, and criminal justice policy. Students learn to focus on the development of mathematical models that can be used to model, improve, predict, and optimize real-world systems. These mathematical models include both deterministic models such as mathematical programming, routing or network flows and probabilistic models such as queuing, and simulation.

Advanced Mathematical Decision Making in Personal Finance

This course is designed to follow the completion of Mathematics I, II, and III. Students will learn how algebra works within the most critical areas of finance. Students will learn about investments, credit, automobile expenses, insurance, income tax, and household budgeting. Students will explore the stock market, banking services, employment basics, budget preparation, planning for retirement, and general independent living. All of these topics will be explored through previously learned mathematics. All students will find the topics of this course useful in their future.

News from the PSC

The Georgia Professional Standards Commission (http://www.gapsc.com/download/PSC_Pulse_2010_Spring_with_links.pdf) has proposed a change in how you can upgrade your teaching certificate by earning an advanced degree. You may now upgrade your certificates by earning an advanced degree in the certificate field you already hold (such as in mathematics or mathematics education if you are currently certified in mathematics). You may also upgrade your certificate by earning an advanced area in a new field, which



must include passing the GACE test and formally adding the field to your teaching certificate. This proposed rule also establishes more rigorous requirements for the institutions from which you can earn advanced degrees in order to upgrade your teaching certificate.

Legislation of Interest

The HOPE Teacher and HOPE Promise teacher scholarships were cut by Georgia's lawmakers towards the end of their 2010 legislative session. For more information on the cut, see <http://www.ajc.com/news/georgia-politics-elections/groups-school-cuts-not-528581.html>. This move was more political than strategic in my opinion. Instead of getting more teachers into Georgia's classrooms and combating teacher shortages, we've made a nearsighted decision that these scholarship programs didn't matter. Senator Seth Harp (R-Midland) was quoted in the Atlanta Journal Constitution as explaining that no one spoke out to keep these scholarships. He said "If you don't beg, you don't get to the table." The many of you who have benefited from these scholarships may want to contact your legislators on the House or Senate Education Committee (http://www.legis.ga.gov/legis/2009_10/house/Committees/education/gahed.htm or http://www.legis.ga.gov/legis/2007_08/senate/education.php)

House Bill 280 was passed in 2009 and became effective last summer. If funded, it provides pay incentives for educators teaching K-5 who successfully complete the K-5 Math or Science Endorsements. To be eligible, you must have at least one full year of classroom teaching experience and be certified in Early Childhood Education, Middle Grades Math or Science or certain areas of Special Education. Nine institutions or RESAS have submitted programs to the PSC for approval. For more information, <http://www.gapsc.com>.

House Bill 1307 was just passed in May and became effective in July 2010. It will "temporarily suspend PLU renewal requirements for a five-year period." The goal was to provide economic relief for educators. For more information, <http://www.gapsc.com>.

Race to the Top (RTTT)

After placing third in the initial round of Race to the Top funding, Georgia decided to reapply in hopes of being awarded up to \$400 million over four

years. Winners will be announced in late August or early September by the U.S. Department of Education. This funding is an effort to support innovation in school improvement through the American Recovery and Reinvestment Act of 2009.

Three additional districts (Dade, Peach, and Pulaski) have joined the original 23 districts involved in the RTTT proposal. Together, they include 41% of the public schools in Georgia, 46% of Georgia's students in poverty, 53% of Georgia's African American students, 48% of Georgia's Hispanic students, and 68% of the state's lowest achieving schools. (Data from Maureen Downey's Atlanta Journal Constitution blog: <http://blogs.ajc.com/get-schooled-blog> on June 1, 2010.)

Update on Common Core State Standards

When you receive this issue of Reflections, you will also be able to go to <http://www.corestandards.org/> and review the final draft of the Common Core State Standards. GCTM was one of the groups that submitted feedback that was used to develop this final draft. I want to recognize GCTM's CCSS Review Task Force members: Linda Crawford, Augusta State University; John Mosely, Bulloch County Schools; Tom Ottinger, retired from Gilmer County Schools and Reinhardt College; Peggy Pool, Atlanta Public Schools; and Debbie Poss, Lassiter High, Cobb County. They reviewed the draft released in March and, on a very tight time line, provided feedback to the Common Core State Standards Initiative writing team. A big thank you to them for providing that service to GCTM.

Because Sony Perdue is the co-chair of the CCSS (which was an initiative of the National Governors Association), the CCSS debuted at Peachtree Ridge High School in Suwanee, Georgia on June 2, 2010. Governor Perdue was there, along with a number of national education leaders.

What are some of the benefits of Common State Standards? In the May 18, 2010 Education Week article titled "Beyond the Rhetoric of National Standards," Gary DeCoker argued that common standards will have the following advantages:



- better articulation between grade levels
- college-entrance exams could be tied to the high school curriculum
- less disruption for students who move
- textbooks would become more coherent and focused
- better textbooks could bring about improvement in teacher's editions
- greater articulation between teacher-education standards and K-12 schooling
- a lessening of local frictions
- a spur to collaboration

What other issues should Georgia's mathematics teachers be paying attention to? Email me at lstallin@kennesaw.edu and I'll include them in the next issue. Have a great summer!

Proposed Change to GCTM Constitution Term of Regional Representatives

Currently:

The regional representatives shall each serve a two-year term. The regional representatives are appointed.

Proposed:

Each region will have two region representatives who will each serve a four year term. One Region Representative will be appointed in year n and the other in year $n+2$.

Rationale:

Adding a region representative and lengthening and staggering the terms of the two region representatives will help with continuity and the potential impact of the region representatives.

CALL FOR *REFLECTIONS* EDITOR!

If you have an interest in becoming the next Reflections Editor, please contact Gregory Chamblee, gchamblee@georgiasouthern.edu for additional information.

Executive Director's Report

by Becky King
Executive Director
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Professional Learning Unit Available From GCTM

Plan Now!

Complete 3 steps and earn one PLU when you attend the 51st Annual Georgia Mathematics Conference at Rock Eagle on October 13-15, 2010.

If you will be attending this year's conference for at least a day and a half, take this opportunity to earn one PLU. You must be able to attend a total of 10 hours of sessions. Please understand that the Georgia Department of Education has a limit of 8 hours of sessions that can be attended on any one day.

The FIRST Step is done in your school system BEFORE you arrive at the Conference.

A Prior Approval Form will be on the GCTM website in August and September. Visit www.gctm.org to download the form. Fill it out, get the required signatures before you leave for Rock Eagle, and bring the form with you.

When you arrive at the conference, pick up your registration materials, and bring the prior approval form to the PLU desk before attending any sessions. You will be given the form for Step 2 and receive instructions for completion of the unit. Step 2 is accomplished at the conference and Step 3 is an on-the-job assessment back in your school system.

The deadline for submission of the on-the-job assessment for the 2009 conference was December 31, 2009. If you have questions regarding this requirement, contact Becky King, Executive Director, GCTM at bwking@comcast.net.

GCTM Mission Statement

The mission of the Georgia Council of Teachers of Mathematics is to

- promote a high quality mathematics education for all students, encourage an active interest in mathematics and in mathematics education,
- promote ongoing professional development for mathematics education, and
- promote and reward excellence in the teaching of mathematics in the state of Georgia.

The objectives of the Georgia Council of Teachers of Mathematics are to encourage an active interest in mathematics and to act as an advocate for the improvement of mathematics education at all levels.

The Georgia Council of Teachers of Mathematics supports and encourages donations to the [Georgia Mathematics Education Trust](#).

GCTM NEWS



Membership Report

by Susan Craig
Membership Director
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3010 Members and Counting

GCTM has begun a second 50 years and has thrown open the doors to new ideas, projects, and membership growth. Our membership is at a relative maximum level of 3010. But as long as there are mathematics teachers in Georgia who are not members or who don't know the wonderful benefits of membership in GCTM, then we need to continue to share the good news with them and encourage them to join us!

What is in store for GCTM members? Professional development opportunities, student activities, excellent journal, superb annual conference, annual mathematics contests, summer workshops, grants for classroom use, honors and awards! All wonderful benefits of GCTM membership, which only get better each year...and all for a small price! The price remains low and the benefits grow!

Help us keep you informed

Go to www.gctm.org, log in, and check the box which will allow us to send you email notices. You must check this option to receive notices to remind you to renew and to allow notices when it is time to vote for officers. These are the only times GCTM will send you emails. Please do this today! It saves postage costs and ensures you are kept up to date on these two important items.

Student Members

If you are a student, it is time to go online and renew your membership. All student memberships expire on June 30th. If you are beginning your career, please consider continuing your membership as a full professional member for only \$20 a year.

If you have any questions please send them along to Susan Craig at secddc@aol.com

Have a wonderful summer respite!



Metro-East Report

by Leanne Luttrell
Representative, Metro East Regional

Hello, Metro East Members!

I hope you are all enjoying your summer and renewing yourself for a great 2010-2011 school year! I, along with one of the GCTM board members, would love to meet with you! There are three purposes for the meeting: to discover how GCTM can best support you, to get to know each other, and to share ideas for teaching mathematics! We will plan to meet on Saturday, August 14 at 9:00 somewhere near Perimeter Mall. (Location will be determined by the number of people attending.) If you are interested in attending, please email me by August 7 (leanne_luttrell@gwinnett.k12.ga.us). I hope to see you there!

Throughout the 2010-2011 school year, please email me with any news, awards, or other accomplishments! I would love to include the information in the regional reports.

High School Mathematics Competition

by Charles Koppelman
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Kennesaw State University Mathematics Competition

The Kennesaw State University Mathematics Competition is an annual mathematics contest open to all Georgia high school students. Unlike contests sponsored by other Georgia Universities, the Kennesaw State University Mathematics Competition is administered to students at their own high school during or after the school day at no cost. In 2009-10, more than 2000 students from 122 public and private high schools competed in round I. Students whose round I scores placed them in the top 10% statewide were invited to compete in round II. Round II consists of five problems each requiring rigorous proof. The students with the top 33 composite scores from both rounds were honored on the KSU campus on April 17.

ALLEN PARK (Walton HS) had the highest composite score in the state and received a \$100 award. CLAUS ZHENG (Walton HS) and OLIVER HUANG (Pace Academy) placed second and third and received awards of \$50 and \$25, respectively.

The following 30 students (in alphabetical order) received certificates of honorable mention.

ABUBAKAR ABID	Walton HS
PARTH AGRAWAL	Chamblee Charter HS
KRISHNAPRASAD AKKINENI	Northview HS
MOHANPRASAD AKKINENI	Northview HS
GRANT CARLSON	Westminster Schools
ERIC CHEN	Brookwood HS
SITAN CHEN	Northview HS
NATHAN CORBIN	Alpharetta HS
KRISHA DESAI	Alpharetta HS
GIL GOLDSHLAGER	Walton HS
ROBERT GROSSE	Chamblee Charter HS
MICHAEL HOPKINS	Pace Academy
BENJAMIN HU	Northview HS
JONATHAN JOHNSON	Rockdale Magnet HS
BHANU KUMAR	Milton HS
JONGHUN KWAK	Tallulah Falls School
JAEWON LEE	Johns Creek HS
NATHAN LI	Northview HS
ANDREI MARKOV	Columbus HS
ARVIND NARAYAN	Northview HS
EDWARD PARK	Walton HS
ANANTH PUNYALA	Alpharetta HS
YUBO SU	Northview HS
PATRICK THOMPSON	Centennial HS
BORU WANG	Parkview HS
EDWIN WHITE	Rockdale Magnet HS
PETER WOOLFITT	Chamblee Charter HS
TIANQI WU	Parkview HS
WELLS YANG	Northview HS
JESSE ZHAO	Chattahoochee HS

Information and registration materials for the 2010-2011 competition will be mailed to the Mathematics Department Chair at each Georgia high school in early September. If you would like more information about the competition, please visit the contest website <http://math.kennesaw.edu/~ckoppelm>.

GCTM NEWS



GCTM 2010 Middle School Mathematics Competition

by Chuck Garner
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Middle School Mathematics Competition

The GCTM Middle School Math Tournament was held at Thomson Middle School in Centerville, GA on April 17, 2010. Middle schools across the state were invited to register up to eight students to compete. The tournament consisted of a 30 question multiple-choice test with a 45-minute time limit; 10 individual ciphering problems, each problem with a two-minute time limit; 3 rounds of four pair ciphering problems (in which students from a school formed teams of two), each round with a four-minute time limit; and a four-person team “power question,” in which the team solves a complex problem with a 10-minute time limit.

The tournament is designed to challenge middle school students and to reinforce classroom skill. However, we also make sure the students have fun! At the conclusion of the tournament, students participate in a fun-only “Frightnin’ Lightnin’” Round, where students must be quick on the draw to answer math problems posed orally. The winners of this round get candy!

The top five teams are below.

TOP TEAMS:

Westminster

Inman Middle School

Greater Atlanta Christian School

Trinity Christian School

Thomson Middle School

Eighty-eight students from fifteen school participated. Sponsors that are members of GCTM only had to pay a \$10 registration fee or submit five multiple-choice questions for possible inclusion in a future tournament. The next GCTM middle school tournament is scheduled for April 23, 2011.

Below are some sample problems from the tournament.

Test Problem #3: Jacob is twice as old as Katie and Katie is 3 years younger than Layla. If Jacob is 5 years older than Layla, then how old is Katie?

- a) 6 b) 8 c) 11 d) 16 e) There is not enough information.

Test Problem #21: Lindsay has a total of \$82.00, consisting of an equal number of pennies, nickels, dimes, and quarters. How many coins does she have in all?

- a) 164 b) 200 c) 328 d) 410 e) 800

Individual Ciphering #4: Solve for x: $-3|x| + 5 = -1$.

Pair Ciphering #2: How many digits are in the number $2^{36} \times 5^{33}$?

Answers: B; E; 2 and -2; 34

GCTM State Mathematics Competition



by Chuck Garner
VP for Competitions
cgarner@gctm.org

State College Mathematics Competition

The 34th annual GCTM State Mathematics Tournament was held at Macon State College in Macon, GA on April 24, 2010. Schools are invited to the state tournament based on their performance on previous Georgia tournaments throughout the 2009-2010 school year. Thirty-three schools were invited to this year's state tournament. Four students are selected to represent each school. Twenty-one individuals were also invited to tryout for the state-wide Georgia ARML team, making a total of 153 participants.

The tournament consisted of a very challenging written test of 45 multiple-choice questions and 5 free-response questions with a 90-minute time limit; 10 individual ciphering problems, each problem with a two-minute time limit; and 2 rounds of four pair ciphering problems (in which students from a school formed teams of two), each round with a four-minute time limit.

The tournament was held with the assistance of the Mathematics Department at Macon State University, as well as the financial assistance of two grants from the Macon State College Foundation and AT&T Georgia.

This year GCTM inaugurated the Steve Sigur Award for Most Improved Performance. This award, named in honor of the great mathematician, teacher, and mentor Steve Sigur, went to Andrei Markov of Columbus High School.

Trophies also went to the top five teams and the top ten individuals. The winners are listed below.

TOP TEAMS:

Walton High School
Parkview High School
Northview High School
Milton High School
Chamblee Charter High School

TOP INDIVIDUALS:

Allen Park, Walton High School
Sitan Chen, Northview High School
Tim Wu, Parkview High School
Edward Park, Dickerson Middle School
Abubakar Abid, Walton High School
Claus Zheng, Walton High School
Robert Grosse, Chamblee Charter High School
Boru Wang, Parkview High School
Oliver Huang, Pace Academy
Eric Chen, Brookwood High School

State Tournament registration is free, but schools must be invited. The next GCTM State Mathematics Tournament is scheduled for April 30, 2011.

Below are some sample problems from the state tournament.

Test Problem #15: A circle that passes through the origin has rightmost point (1, 7). Compute the radius of this circle.

a) $2\sqrt{6}$ b) 6 c) $5\sqrt{2}$ d) 8 e) 25

Test Problem #27: How many positive integers have their digits in strictly increasing order?

a) 18 b) 511 c) 896 d) 181440 e) Infinitely many

Test Problem #41: The equation $\cos[k\theta] - \cos[(k+2)\theta] = 0$, where k is an integer, has 96 solutions on the interval $(0, 2\pi]$. Compute the sum of the digits of k .

a) 5 b) 11 c) 12 d) 15 e) 16

Individual Ciphering #3: Points A and B divide a circle into a major and minor arc. The inscribed angle of the major arc equals the central angle of the minor arc. Find the degree measure of the inscribed angle of the minor arc.

Answers: E; B; B; 60

GCTM NEWS



ARML 2010

by Chuck Garner
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American Regions Mathematics League (ARML) Tournament



The 35th annual American Regions Mathematics League (ARML) tournament took place at UGA on Saturday, June 5. The Georgia ARML “A1” team finished 14th in the nation in the A division and the Georgia ARML “B” team finished 17th in the B division. This marks the 21st straight year that the Georgia team finished in the top

10% at the “world series” of math team tournaments.

Held simultaneously at four sites around the nation (Pennsylvania State University, University of Nevada at Las Vegas, University of Iowa, and University of Georgia), this competition attracts over 2000 students from all over the U.S. as well as Canada, Taiwan, China, Colombia, and Vietnam. Any geographically contiguous region may enter as many 15-person teams as it likes, whether that region is as small as a school district or as large as a state, as long as regions do not overlap. In Georgia, three teams of 15 (plus alternates) are selected each year based on results from the state tournament, AMC scores, AIME scores, and winning scores at other Georgia tournaments. The Georgia A1 and A2 teams are made of up experienced ARML participants, while the Georgia B team is for promising newcomers to ARML.

The ARML tournament brings together the nation’s finest students, where they meet, compete, and socialize, forming friendships and sharpening their mathematical skills. The contest is written for high school students, although some highly-talented middle school students attend each year.

The tournament consists of four rounds. The team round is first, in which the 15 teammates have 20 minutes to solve 10 problems. Next is the power round, in which the team has 60 minutes to write proofs to complex, multi-part problems. This year’s power round topic involved proving statements concerning Descartes’ Circle Formula. After this is the only round that is not team-oriented, appropriately called the Individual Round. For this round each student solves 10 problems independently. However, the students receive the problems two-at-a-time with a time limit of 10 minutes per pair of problems. After a break is the final round, the relay round. The 15 teammates form five relay teams of three each, where one person’s answer is used in the next person’s problem.

National sponsorship of ARML is mainly provided by the D. E. Shaw Group, an investment and technology development firm. Other national sponsors include the American Mathematical Society, Art of Problem Solving, Key Curriculum Press, Math League, Mu Alpha Theta, Texas Instruments, and Wolfram Research. Locally, sponsorship of the Georgia ARML teams is mainly provided by GCTM.

Coaches of this year’s Georgia ARML team were Tom Fulton (Walton High School), Chuck Garner (Rockdale Magnet School), Henry Oglesby (Creekview High School), Debbie Poss (Lassiter High School), Carol Sikes (South Forsyth High School), and Head Coach Don Slater (Lassiter High School). Joining the coaching staff this year were two former Georgia ARML participants: Harrison Brown and Santhosh Karnik, both currently students at Georgia Tech.

The members of the outstanding 2010 Georgia ARML team are listed below.

AbubakarAbid	Walton High School
Moncia Agrawal	Chamblee High School
Jonathan Austin	Northview High School
Shivani Baisiwala	Walton High School
Udai Baisiwala	Dickerson Middle School
John Calhoun	Pace Academy
Grant Carlson	Westminster
Slok Carpenter	Walton High School
Gautam Chebrolu	Columbus High School
Eric Chen	Brookwood High School
Ryan Chen	Crews Middle School
Sitan Chen	Northview High School
Lilly Chen	Westminster
Alex Commanday	Milton High School
Nathan Corbin	Alpharetta High School
Ryan Dickmann	Columbus High School
Andrew Ding	Augusta Prep
Val Felso	Soli Deo Gloria Homeschool
Gil Goldshlager	Walton High School
Robert Grosse	Chamblee High School
Alex Harrington	Creekview High School
Michael Hopkins	Pace Academy
Ben Hu	Northview High School
Oliver Huang	Pace Academy
Jonathan Johnson	Rockdale Magnet School
Heeyoon Kim	Rockdale Magnet School
Kyung Kim	Northview High School

Mathematics Competition



Bhanu Kumar	Milton High School	Boru Wang	Parkview High School
Michael Lai	Athens Academy	Kevin Wen	Cedar Shoals High School
Andrei Markov	Columbus High School	Edwin White	Rockdale Magnet School
Mark McDonald	Creekview High School	Tim Wu	Parkview High School
Sophie Mori	Dodgen Middle School	Jiyang Xie	Roswell High School
Arvind Narayan	Northview High School	Anchen Yao	Parkview High School
Allen Park	Walton High School	Ksenia Zakirova	Rockdale Magnet School
Edward Park	Dickerson Middle School	Luna Zhang	Wheeler High School
Anath Punyala	Alpharetta High School	Jesse Zhao	Chattahoochee High School
Aaron Roberts	Rockdale Magnet School	Claus Zheng	Walton High School
Sneha Sareedy	Wheeler High School		
Roger Song	Walton High School		
Yubo Su	Northview High School		
Anirudh Tadanki	Milton High School		

Below are some sample problems from this year's ARML tournament.

Team Round Problem 9: The equations $x^3 + Ax + 10 = 0$ and $x^3 + Bx^2 + 50 = 0$ have two roots in common. Compute the product of these common roots.

Power Round Problem 3b: [Note: Curvature is defined as the reciprocal of the radius. Given four pairwise tangent circles of curvatures a , b , c , and d , then Descartes' Circle Formula says that $(a + b + c + d)^2 = 2(a^2 + b^2 + c^2 + d^2)$.] Given three mutually tangent circles with curvatures a , b , $c > 0$, suppose that a , b , c , and 0 do not satisfy Descartes' Circle Formula. Prove that there are two distinct values of r such that there is a circle of radius r tangent to the given circles.

Individual Round Problem 3: Points P , Q , R , and S lie in the interior of square $ABCD$ such that triangles ABP , BCQ , CDR , and DAS are equilateral. If $AB = 1$, compute the area of quadrilateral $PQRS$.

Relay Round 2 Problem 1: A fair coin is flipped n times. Compute the smallest positive integer n for which the probability that the coin has the same result every time is less than 10%.

GCTM Grants and Awards Opportunities

- Gladys M. Thomason Award for Distinguished Service
- Dwight Love Award
- John Neff Award
- Awards for Excellence in the Teaching of Mathematics (Elementary, Middle & Secondary levels)
- Teacher of Promise Award
- Mini-Grants
- Special Projects

For additional information visit the GCTM website www.gctm.org.

GCTM NEWS



Georgia Mathematics

Theme — “Mathematics: Connecting the Pieces

The 2010 Georgia Mathematics Conference will be held October 13-15, 2010 at the Rock Eagle 4-H Center outside of Eatonton, Georgia.

Some Featured Speakers/Workshops

● WEDNESDAY

- Wednesday afternoon Open House Sessions have been changed to “Share Shop.” There are four sessions in different rooms.
- We will have two large game rooms
 - ◆ Room One will be a place to try out math games called “BYOG (Bring Your Own Game).” Lynn will facilitate possibly with the assistance of some MAT students from Kennesaw State University. Carole Tilley with Educators Outlet will provide some games but teachers are encouraged to bring their own to share. These games can be ones they purchased or created themselves.
 - ◆ Room Two will be the “Game Tournament” room.
- We will have two large rooms for crafts called “Make-It & Take-It” rooms.
 - ◆ Room Three will have craft stations with supplies and facilitators for grades 9-12 stations.
 - ◆ Room Four will have craft stations with supplies and facilitators for grades PK-8 stations.
 - ◆ Teachers are encouraged to come learn some crafts to make with their students or bring some they can teach others.

● THURSDAY

- **Greg Tang**--author of books including **The Grapes of Math**, **Math For All Seasons**, **The Best of Times**, **Math Appeal**, **Math-terpieces**, **Math Fables**, **Math Potatoes**, and most recently, **Math Fables Too**. He hopes that his books will give children a better understanding of math, from counting all the way through calculus. Today, Greg is working to develop a more intuitive approach to teaching math, one that combines problem-solving and arithmetic and integrates math with language and art. In addition to writing his picture books, Greg is also an author for Houghton Mifflin Harcourt’s new textbook programs in California, Texas, the Carolinas, and Kentucky. He wrote the Go Fast Go Far™ mental math series and is creating a line of teaching materials that includes flash cards, math games, and workbooks. His math games **Numskill™** and **Numskill Jr™** have been an instant hit with teachers and students, and his **Smart Sums™** and **Great Times™** flash cards have been widely praised by both teachers and parents alike.
- **Karen Norwood**--MINDSET: Tools (MINDSET) is a collaboration between educators, engineers, and mathematicians at three universities to create, implement, and evaluate a new curriculum and textbook to teach standard mathematics concepts using math-based decision-making tool. Participants will receive an overview of the project, discuss the ongoing research efforts, and have an opportunity to experience the curriculum through problem-solving.

● FRIDAY

- **Ron Lancaster**--“The Mathematical Lens,” 6-12, general interest. We may not realize it, but we all encounter mathematics as we go about our lives. To see the mathematics it helps to put on a pair of mathematical glasses and to view the world through this new math by taking photos and how these images can then be used as the basis of interesting and engaging mathematical questions. We will also discuss how the Geometer’s Sketchpad can be used as a tool to answer these questions.
- **Dorothy White**--K-5 workshop, Title, Topic, TBA.

Conference 2010



Keynote Speakers

Steven Leinwand - Keynote
Wednesday
October 13, 2010



TITLE: Some Intriguing, Amazing and Even Scary Things We Can Learn from High-Performing East Asian Countries that Provide Mirrors on How We Teach Math in Georgia.

DESCRIPTION: It's really not an accident that countries like Singapore and Hong Kong significantly outperform the U.S. As examples of "international benchmarking," this session will take a look at a range of curriculum features, instructional approaches, and assessment items drawn from these countries that provide intriguing insights into how the U.S. might ratchet up how we teach and how we assess mathematics.

BIOGRAPHY: Steve Leinwand is a Principal Research Analyst at the American Institutes for Research (AIR) in Washington, DC and has over 30 years of leadership positions in mathematics education. He currently serves as mathematics expert on a wide range of AIR projects that evaluate programs, develop assessments, and provide technical assistance. Leinwand's most recent work has included the development of test and item specifications and an Algebraic Reasoning item pool for the NCES High School Longitudinal Study and serving as Implementation Task Leader for the IES Mathematics Professional Development Impact Study. His work, supported by a task order from the U.S. Department of Education, resulted in "What the United States Can Learn from Singapore's World-Class Mathematics System (and what Singapore can learn from the United States)" and more recently a comparison of the 2007 Grade 3 assessments administered in Hong Kong and in Massachusetts. As part of AIR's assessment program, Leinwand has overseen the development of multiple-choice and constructed response items for AIR's contracts with Ohio, Hawaii, and South Carolina.

Before joining AIR in 2002, Leinwand spent 22 years as Mathematics Consultant with the Connecticut Department of Education where he was responsible for the development and oversight of a broad statewide program of activities in K-12 mathematics education including the provision of technical assistance and professional development, the evaluation of Title 1 and K-12 mathematics programs, the assessment of student achievement, teacher competency, the dissemination of information, and the coordination of programs and activities. Steve has also served on the NCTM Board of Directors and helped with the development and publication of Principles and Standards for School Mathematics, been President of the National Council of Supervisors of Mathematics during a period of increasing demands on state, district, and school level mathematics leaders, and served on the Mathematical Sciences Education Board during the development and publication of "Everybody Counts."

Steve is also an author of several mathematics textbooks. He has written numerous articles. His books, *Sensible Mathematics: A Guide for School Leaders* and *Accessible Mathematics: 10 Instructional Shifts That Raise Student Achievement* were published by Heinemann in 2000 and 2009 respectively. He continues to balance his work at AIR with periodic seminars on "Making Math Work for Underachieving Students" including the development of instructional strategies for improving student achievement generally, and in the area of fractions, decimals, proportional reasoning, measurement, and estimation specifically.

Among his interests are alternative assessments, reform strategies, developing and supporting change agents, and leadership strategies for overcoming obstacles to change.

GCTM NEWS

Ron Lancaster- Keynote
Thursday
October 14, 2010



TITLE: The Housekeeper and the Professor by Yoko Ogawa: Using Fiction and Film to Teach Mathematics.

DESCRIPTION: The focus of this talk will be on *The Housekeeper and the Professor*, a novel by Yoko Ogawa, that tells a touching story about memory, family, and a boy with a flat head named Root who grows up to be a mathematics teacher. A great deal of mathematical content is woven into the story in a highly engaging manner. Mathematical moments from the *Professor's Most Beloved Equation*, a movie based on the book, will also be discussed. This talk will appeal to teachers looking for ways of incorporating fiction and film into their courses or to anyone looking for a great book to read and film to enjoy.

BIOGRAPHY: Ron Lancaster taught middle and high school mathematics for over 20 years and worked full-time as an independent mathematics consultant for schools, educational organizations and technology companies in North America, Asia and Israel for six years. He is now a Senior Lecturer in mathematics education at the Ontario Institute for Studies in Education of the University of Toronto

Ron has been a presenter at hundreds of conferences, including the highly-regarded Phillips Exeter Academy Mathematics and Technology Conference for the past nineteen years; 35 NCTM (National Council of Teachers of Mathematics) Annual Conferences; 27 consecutive OAME (Ontario Association for Mathematics Education) Annual Conferences and 6 EARCOS (East Asia Regional Council of Overseas Schools) Conferences. The topics for his workshops address a wide range of issues such as connecting mathematics with art; the use of photos as a way of promoting mathematical enquiry; and the use of handheld technology and computer software. Ron is well known for his expertise in designing Math Trails and has conducted numerous workshops for teachers on this topic. Ron has created these math paths in many cities, particularly in Singapore where over 7000 students and hundreds of teachers have enjoyed his walks in shopping centres, parks and art museums. Ron has developed a number of Math Trails in Manhattan for Math or America at locations that include the Museum of Modern Art; The Museum of Natural History; Ellis Island; the Bronx Zoo; Madison Square Park and the campuses of NYU and Columbia University.

Ron has been a National Instructor for Texas Instruments since 1994 and has conducted dozens of week-long and day-long workshops through the US and Canada on graphing calculators.

Ron was an on-air teacher and consultant for several television series, one of which won a Gold Medal at the 1990 International Film and TV Festival of New York.

Ron was a member of the grades 9-12 Writing Team for the NCTM's *Principles and Standards for School Mathematics* published in 2000. Ron created two columns (*Media Clips* and the *Mathematical Lens*) for the *Mathematics Teacher* published by the NCTM. Ron edited *Media Clips* for three years and continues to edit the *Mathematical Lens*. Both columns have a huge following with readers.

Ron is the only teacher in Canada to have been honored with three prestigious awards for innovative teaching from the Hilroy Foundation and he has won other awards for his creative and engaging style of teaching.

Ron Lancaster- Keynote
Thursday
October 14, 2010



TITLE: Embracing the Cultures in Every Mathematics Classroom to Support Learning

DESCRIPTION: TBA

BIOGRAPHY: Dorothy Y. White is an Associate Professor of Mathematics Education at the University of Georgia. She teaches undergraduate and graduate courses for preK-8 preservice and inservice teachers. Her research focuses on equity and culture in mathematics education, developing models of productive collaborative mathematics planning, and the professional development of urban mathematics teachers.

Be-Good-To-Your-Partner Tennis: Lessons to Help You Teach Better Mathematics (and this is not an article about parabolas!)



My husband and I play an odd version of tennis. In fact, we probably shouldn't call it tennis. Many of the rules related to a regulation game of tennis don't apply. Our objective is to get exercise, so we have devised a way to make that the first priority. We don't stop to serve or change courts or get behind the baseline because that is down time, and we don't keep score because there is no score to keep. We are trying to keep moving, so we hit any ball we can get to—out of bounds or in, one bounce or two or five. We start with a couple of balls each, and when one goes into the net, we pull out another, volley it right away, and keep going. If a ball goes into the back fence, we go after it and volley it back over.

One of the things that keeps us moving more than anything is that we try to actually hit it TO each other rather than away. For this purpose, it is advantageous to Be-Good-To-Your-Partner, and both of us benefit. It seems that there are many parallels between being good at playing Be-Good-To-Your-Partner Tennis and being good at teaching mathematics. Here are some rules that may help to make you successful at both.

1. **Watch what leaves you and goes back to your partner; watch it intently all the way.**

In tennis, this is how you gauge where your partner will be next and where you will place your next shot. You don't just hit it and turn away; you need to watch to see where it landed—whether it went into the net, whether it went over the fence, whether it looks like your partner will be hitting it back, how he will be approaching the shot, whether he must lob it, or if he has a chance to slam it toward you. We also learn how effective our shot was.

In teaching mathematics, we don't just need to watch what students send to us. We need to carefully watch what leaves us and goes to them. We cannot just volley a concept out there and then begin immediately thinking about the next problem we will pose. There are two reasons why it is important to watch their immediate responses to what we say:

- ⟨ We can more clearly predict what will come back from them and can handle it better.
- ⟨ We are more reflective practitioners—what did WE do and how did it work?

I have often told teachers to “teach to the wrinkled eyebrows.” When you laid out a definition, were there lots of wrinkled eyebrows? When you asked a question, did 75% of the students look down, hoping you would not call their names? Did you see lots of understanding in their faces and know that you could move on? Watch intently as your volleys go from you to your students; you can definitely improve your game.

2. **Watch what is coming toward you; watch it intently all the way.**

In tennis that means that you watch the ball as it is coming to you. Visualize a tennis ball coming across a net toward you. Better than that, have someone bounce a ball to you. Did you keep your eye intently on the ball? Our son is a baseball coach, and “eye on the ball” is a key to good hitting. Have someone bounce the ball to you again, and this time, watch the ball **intently**. You will see something very different from the first time. Watching intently makes the ball almost appear to be in slow motion. After it bounces, you will be able to see clearly the direction the ball is taking and when it reaches its apex. If you are preparing to respond to that ball, a good response depends totally on knowing what is coming to you, so keeping your eye on it all the way is incredibly important.

In teaching mathematics, we often concentrate on what we say, but not necessarily on what is being said to us. Spend a day **intentionally listening** carefully to what your students are saying and doing in response to you. Focus on their facial expressions and their words, and you will do a better job of responding to their ideas and misconceptions. There is an additional gain—your students will know that you are sincerely interested in their responses and questions.

3. Know your partner.

In tennis, especially the way we play it, you have to know your partner's strengths and weaknesses. My partner has a really strong forehand and he can put a spin on it. I get back good strong returns when I send the ball to his forehand. He also has two bad knees and wears braces on both when we play. That means he can't lunge or run far or change directions quickly. If I want to keep getting exercise, I have to plan to hit the ball right to him. If he were more mobile, I would work to allow him to stretch his capability. In turn, he knows me, and he concentrates on getting me to run to get the workout I want.

In teaching mathematics, we have to know our students. Our job is to know more about their mathematical ability and learning styles than they do. Be sure that, when you think about learning styles, you aren't limiting yourself to considering the basic auditory, visual, and kinesthetic ideas. Do some of your students need multiple guided and independent practice problems? Do some of your students understand a procedure better when they work through it with a partner? Do some need to write down numbered steps to solving a particular type of problem? Do some of your students only understand a mathematical idea when it is grounded in reality? Do some of your students prefer to work using mnemonics?

Yes, we help them work through their weaknesses, but sometimes I have found myself only concentrating on weaknesses. For every struggling student in your class next fall, I challenge you to determine **at least one strength**, maybe more. Then—and this is the key—communicate to the student that you have identified that strength, and that you want to build on it.

4. Really determine to be good to your partner; let that be your daily mind-set.

In the kind of tennis I play, I truly must go out there every time with a determination to be good to my partner, and that mind-set is totally different from the competitive mind-set that is necessary for regular tennis. If I want to continue running and gunning, I need to hit the ball where my partner can return it. It can challenge him a bit, but not too much. I have to keep in mind that his success determines my success. For us both to be successful, I have to have his best interest at heart.

In teaching mathematics, we have to be good to our students. We have to have their best personal and mathematical interests at heart. It is not a competition; it has to be a partnership. Granted, sometimes making a partner out of an unwilling student is challenging, but we still cannot change the basic rule: Be good to your partner.

Not only do we have to “put it where they can get it,” but we have to treat them well. Teachers find many ways not to treat their students well. I have been in classrooms where the Golden Rule was no part of the culture. Sometimes teachers use sarcasm, sometimes they use yelling, and sometimes they use total

indifference. You may have students who find the mathematics challenging, but one thing they will understand immediately is whether their teacher cares that they learn or not. I am certainly not saying that you should not run a tight ship. One of the best ways to work with struggling students is to have clear rules and procedures that you always enforce.

Especially when teaching today's challenging curriculum, we must keep in focus that it is possible to make even challenging mathematics accessible to students. We recently lost a great mathematics educator, Jaime Escalante. That news prompted me to watch again *Stand and Deliver*, the movie of his life. How different are the struggling children in your classrooms from those he taught? Look where he took them, and he didn't just do that once; he did it year after year, changing the lives of those children and the culture of a whole school. It reminds me of a quote that I love from Ron Edmonds: “We can, whenever and wherever we choose, teach all children whose education is of importance to us.”

5. Be prepared every day with a daily plan.

In tennis, you have to prepare for the game. I have to come to the tennis court with a towel, water, tennis racquet and balls, and a watch. I also have a planned allocation of time. If I don't prepare for the game, I won't get the most out of it. I also have to prepare for each swing. If I don't position myself and get the racquet pulled back, I won't get the best shot, and it won't have the most power.

In teaching mathematics in Georgia, I am seeing an odd trend away from planning, especially by the lesson. Instead, I see teachers using a unit plan as an overall plan, and they let each day follow the previous one in the unit. The result is daily lessons that do not have a clear focus, and the outcome is less than stellar.

The truth is that learning doesn't happen by the unit; it happens by the class period—it happens in that time between “hello” and “see you tomorrow,” and if it isn't planned that way, you are just taking potluck. For that reason, lessons must then be designed by the period. You as the teacher, are responsible for deciding—not what they will know at the end of a unit, but what they will know at the end of the class period that they did not know when they walked in the room. That period is the time that you are working with, and that time period defines how your plan must be written. And—WRITTEN is a key. Design a lesson, beginning with what you want them to be able to do when they leave, and then plan everything from the housekeeping items that take place at the beginning to the way you will focus their attention on the new learning, through every example, every practice problem, every real-life application, to the way you will know whether they have learned. Don't forget to plan for probing questions and for that last special interesting math idea you'll share that takes them to the very second that class is over.

6. Concentrate to get in the zone.

In tennis, you have to stay in the game. If a plane flies overhead and you look up to see it, you miss the ball. If your eye focuses on a leaf on the court or on someone who walks by, you lose your concentration. When you lose your concentration, you lose.

When you lose your concentration in the classroom, students lose their focus and both of you lose because learning and retention are reduced. When you enter that area of complete focus in sports, it is called being “in the zone.” The same thing can happen in teaching, and a teacher knows when it happens. You can feel it. Concentrate to get in, and stay in, the mathematics zone.

Perhaps applying all of these Be-Good-To-Your-Partner rules can help you—and your students/partners—too get in the zone and to reach high levels of success in teaching and learning mathematics at all levels.

Here are the Be-Good-To-Your-Partner rules for teaching mathematics in a list:

1. Watch what leaves you and goes back to your partner; watch it intently all the way.
 - ⟨ Watch their expressions carefully to check for understanding and misunderstanding.
 - ⟨ Use these observations to know how to proceed in your lesson and also to improve your practice in the future.
2. Watch what is coming toward you; watch it intently all the way.
 - ⟨ Listen intently to their answers to your questions.
 - ⟨ Look carefully at what they write and how they proceed with a problem or task.
 - ⟨ Use that knowledge to calculate your next move.
3. Know your partner.
 - ⟨ Know your students’ weaknesses, but more importantly, know their strengths - - including those strengths that may not have anything to do with mathematics.

4. Really determine to be good to your partner; let that be your daily mind-set.

- ⟨ Begin every class with the mind-set that it is in your and their best interest that they learn some good mathematics today.
- ⟨ Have clear rules and procedures; then calmly and positively stick to them.
- ⟨ Put the mathematics within their reach. Remember Jaime Escalante. You can do that, too.

5. Be prepared every day with a daily plan.

- ⟨ Learning happens by the class period, so plan by the period. Have a written plan for every day that includes some mixed review at the beginning, an activating strategy, which practice problems you will use, etc. Do not wing it. You end up looking worse than a bridesmaid in a wedding chewing gum.
- ⟨ Be absolutely sure that you have decided what they will know when they leave your class that day that they didn’t know when they got there.
- ⟨ Include real-life applications as much as is humanly possible. It makes them trust you.

6. Concentrate to get in the zone.

- ⟨ Focus at the beginning of class on what your students are to learn TODAY. Then move through your lesson plan with that objective in mind.
- ⟨ Be sure that they also know what the goal is so they will know when they have achieved it. The “I’ve got it” moment is important in their personal assessment of learning.

(Based on part of a workshop given September 2009 to the faculty of the Mathematics Department at Abraham Baldwin Agricultural College, Tifton, GA)

Learn More About Your Organization

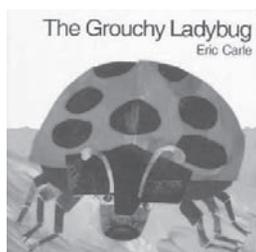
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Judy's Bookshelf

by Judy Chambers
Independent Math Consultant
judychambers100@hotmail.com

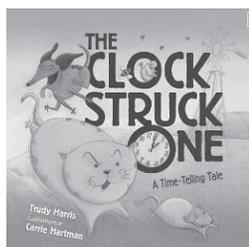
Are you looking for a good math literature book to help introduce your math vocabulary? Or maybe you want to demonstrate how a concept relates to real life. Do you need a good lead-in to the concept? There just seems to be an endless supply of math literature books out there. And I love sharing them with you. I've said it before and I'll say it again, nothing starts a math lesson any better than a math literature book. It also gives you a chance to introduce and reinforce math content vocabulary. Books can also serve as a springboard for writing activities. But enough of that, let's look at some books.



Telling time is a skill that appears and reappears over and over throughout elementary school. Telling hours, telling minutes, parts of the day, elapsed time, and on and on. I've found several books; some old, some new, that I think will enhance the telling time lessons, regardless of the grade level. One of the

top choices for fun and learning is Eric Carle's *The Grouchy Ladybug* (1977 HarperCollins). For starters this book is big and colorful. The pictures are very inviting for children. As you read this book, it talks about the time and what the ladybug is doing at each hour. And on each page, there is a small clock in the upper corner that shows the time mentioned on the page. By the time you've gone a couple of pages the format changes and the page gets small but continues to grow as you go through the book and the hours of the day until you reach 5:00 again and the page is full size again. There are other surprises in the book to delight the children as well. This book is great for working on hours only or for beginning elapsed time with hours only. For a writing follow-up, students could make their own flip book with activities in their day and the time they occur. And, for you sewing enthusiasts, did you know that there is fabric licensed by Carle for this book and several others from his collection. I've seen some great quilts using them. Or you could use a yard to decorate a bulletin board during your time unit. Whatever you choose to do, this is a great book to start your lesson or unit.

Another good book for time to the hour is *The Clock Struck One* by Trudy Harris (2009 Millbrook Press). It's large, colorful and starts off on page 2 with "Hickory, dickory, dock, a mouse ran up the clock. The clock struck ONE, He said, "What fun!" (But the



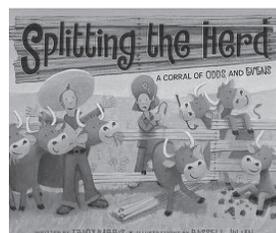
mouse was in for a shock.) and it gets funnier from there. The illustrations are interesting and bright. Carrie Hartman did a great job with this book. For younger students, it's a good book for time to the hour. Re-read the story and let students listen with a student clock in hand. Have them show you the time as the story progresses. Older students can take the nursery rhyme variation and try to write their own version and illustrate it. Whatever you choose, it's a book that deserves a spot in your telling time collection.



For older students, there's a new book out entitled *The Kung Fu Puzzle, A Mystery with Time and Temperature* by Melinda Theilbar (2010, Graphic Universe, a division of Learner Publishing Group). This book has all the elements that attract upper elementary and middle school students: cartoon format, Kung Fu, and a mystery. And it's about time and temperature. This book is one of four from

the Manga Math Mysteries series. The story is good and the students will enjoy reading the book. There are plenty of time and temperature references in the story so you can initiate lots of math discussion in the class.

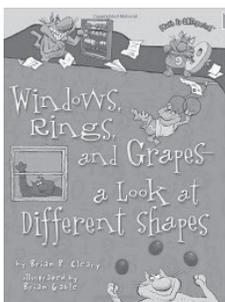
The last book on time that I would like to mention is directed more toward upper elementary and even middle school as it talks about time, calendars, Chinese calendars, clocks, watches, famous timepieces, nature's time, time sayings, etc. But it is interesting background information from a historical standpoint. The book is entitled *Keeping Time* by Avelyn Davidson. (2003, Rigby) It's part of Rigby's InfoQuest series.



Another new book I want to share with you this time is *Splitting the Herd: A Corral of Odds and Evens* by Trudy Harris (2008, Millbrook Press). The book is primarily for ages 5-8 and deals with odd and even numbers. Harris has written the book in rhyme and along with the cute

illustrations makes this an enjoyable book for the class. Its set on a farm and Emma's cows escape and head to Cowboy Kirby's to eat his hay. They then work out a plan for separating

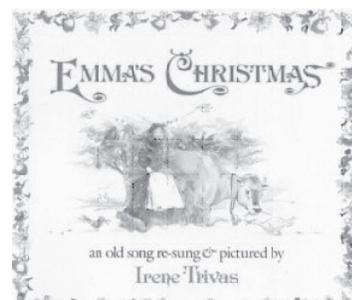
the cows and getting them where they belong. The students will get plenty of practice counting to 20 and identifying even and odd numbers. Emma and Kirby encounter problems along the way but solve it with a very happy ending. The last page of the book offers some discussion questions for follow-up. Young students could have fun drawing the cows and illustrating parts of the story. Some could even write their own version and how they would solve the problem. This is a cute book and definitely deserves a spot on your bookshelf.



In my last column I talked about Brian Cleary's set of books from the Math is CATegorical series. One I did not mention is *Windows, Rings, and Grapes—a Look at Different Shapes* (2009 Millbrook Press). This book, like the others, stresses math vocabulary using two very comical cats and some outlandish illustrations. This whole series is wonderful, especially for developing vocabulary and mental

pictures of math terms. Plus, the pictures provide a good laugh for the students. The book is written in rhyme and the text is scattered all over the page, making it even more charming. Cleary also offers activities, games, and more at his website, www.briancleary.com. Along with his Math Is CATegorical series of 5 books, he also has Words Are CATegorical books for those of you that are self contained and are looking for books for grammar and parts of speech. Follow-up activities can be just as much fun as the books themselves and will help develop the students understanding of math terms.

Now I know its springtime now but I also know that teachers like to plan ahead. Here's one for you to check out, especially if you look for ways to integrate Christmas and Math. *Emma's Christmas* by Irene Trivas (1992, Orchard Books) is a variation on the



Twelve Days of Christmas and involves a little girl named Emma. For younger children it's a counting book or a number pattern book. For older children it's a great way to explore the formula for the sum of consecutive counting numbers. Have them create a table for each day. Look at the number pattern created each day. Let them extend that pattern to the twenty days of Christmas and illustrate it. For students in the upper elementary grades, let them create a graph showing the number of gifts Emma gets each day. Even in middle school, students could use this story to explore and construct triangular numbers. For writing, have students write a new ending to the story or recreate the story using different gifts each day. While this book is written as a variation of a Christmas carol, the book is quite humorous and one the students would really enjoy.

I hope you've found something new to add to your collection. If you have a new book you'd like me to share with others, please email it to me for inclusion in future columns. In the meantime, I will look for new books to share with you in the fall. Enjoy your summer and remember, keep reading that math!

Call for Manuscripts

Topics:

GPS implementation manuscripts are needed. For example, instructional strategies to teach GPS, GPS implementation issues, working with special populations in a GPS environment and sample student task solutions are some of the ideas of interest.

Teaching Tips Ideas:

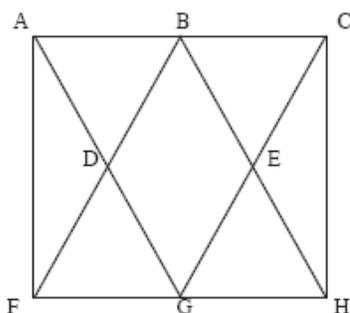
Share with your fellow teachers a pearl of instruction or assessment wisdom you have used in your classroom. Topics include how to design and implement effective warm-ups, strategies for implementing journal writing, etc. Manuscripts published in this section are typically one page in length.

Assumptions and Possibilities: Part Two

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Assumptions and Possibilities (Part 2): 8th Grade Tasks in High School

In a previous Reflections article, 'Assumptions and Possibilities: A Task from 8th Grade Mathematics (54(1), 18-20), we presented two extensions of the Window Pain Task from the Georgia Mathematics Grade 8 Framework that is summarized below.



Window Pain Task: Your best friend must replace a broken window. The window is rectangular, has seven panes, and looks like the diagram above. The window company representative said they only need the measurements of $\angle BAD$ (60°), $\angle BCE$ (60°), and \overline{AG} (28 inches) in order to make a replacement window. Explain in detail how to find every angle measure and every edge length for each pane using the given information.

The new problems were obtained by modifying the assumptions of the original task. The first problem we posed suggested that the figure was not symmetrical about the line BG. As we will explain, this modified problem addresses the 8th grade Georgia Performance Standards (GPS) for Geometry (M8G1 and M8G2) and, with a slight variation, it addresses a geometry standard from the GPS for Math 2 (MM2G3). The second modified problem stipulates that BEGD is a rectangle. In a similar way, we offer two variations of this problem and their solutions, explaining how the first variation can be solved using the mathematics described in 8th grade GPS whereas the second requires concepts from high school GPS. These two problems and their variations illustrate our main point: small changes in the statements of problems can alter to a great degree the

mathematics needed to solve them. In conclusion, we briefly discuss the implications of this fact for students and teachers of mathematics.

In the statement of the first problem we do not assume that the window is symmetrical around the line BG. In particular, this means that B may not be the midpoint of AC. We do suppose (as

in the original problem) that ACHF is a rectangle, that $\triangle ABD$ is equilateral, and that the length AG is 28 inches. In addition, suppose that $\angle AGC$ and $\angle FBH$ are right angles. To arrive at a solution using only 8th grade mathematics (as defined by the GPS), it is sufficient to draw three vertical auxiliary lines (Figure 1): from D to M (the midpoint of AB), from G to B, and from E to N (the midpoint of BC). The Pythagorean theorem, used with the fact that $AD = 14$ inches and $AM = 7$ inches, gives $DM = EN = 7\sqrt{3}$ inches and so by the property of similar triangles (M7G3),

BG is equal to $14\sqrt{3}$ inches. Since $\triangle BEG$ is an equilateral triangle, $BE = BG$, and the Pythagorean theorem can be used to find that $BN = 21$ inches. The rest of the values for the remaining lengths and angle measures are straightforward to compute using the Pythagorean theorem (M8G2), the properties of angle pairs formed by parallel lines cut by a transversal (M8G1b), and fact that two triangles are congruent when all corresponding angles and sides are congruent (M8G1d). Since similar triangles are introduced in grade 7 and the Pythagorean Theorem is introduced in grade 8, this new problem is appropriate for the 8th graders.

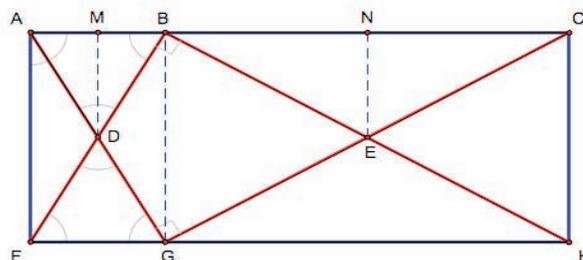


Figure 1. The auxiliary lines MD, BG, and NE

A slight variation makes this problem appropriate for Mathematics 2. Retaining the original stipulations that $\triangle ACHF$ is a rectangle and that $\angle AGC$ and $\angle FBH$ are right angles, suppose that only $\angle BAD$ is 60° instead of assuming that $\triangle ABD$ is equilateral; this means in particular that the measure of $\angle ABD$ is not known. If $\angle ABD$ is unknown, then students might think at first that there could be an infinite number of solutions to the task of identifying all of the side lengths and angles in the diagram (one for every possible measure of $\angle ABD$). In fact, there are only two solutions; they correspond to the two positions for point B indicated by an auxiliary semicircle with the diameter FH (Figure 2). Since an inscribed angle measures half its intercepted arc (MM2G3c), this semicircle must pass through all possible positions of point B where $\angle FBH$ is a right angle. These positions include the one discussed in the first variation of this problem (when the measure of $\angle ABD$ is 60°) and the one indicated by point B' (the measure of $\angle ABF$ is 30°). Students must be familiar with content from Math 2 to recognize this fact and to know as a consequence that there are only two solutions to the problem of finding all the measures of segments and angles in the figure.

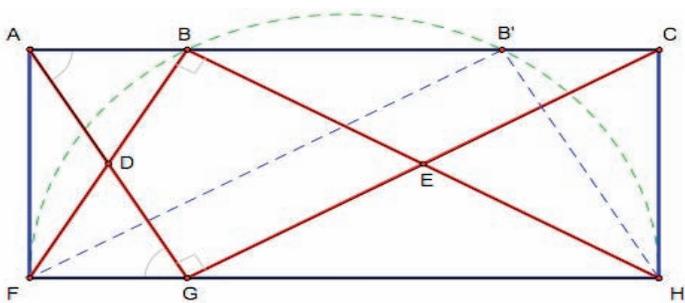


Figure 2. A semicircle with diameter FH passes through point B (and B').

The second problem obtained as an extension of the Window Pain Task assumes that $\triangle ACHF$ and $\triangle BEGD$ are rectangles, AG is 28 inches, and $\angle FAD = 60^\circ$. In the first variation an auxiliary line LD is given, such that point L is on AF and $\angle ALD = 60^\circ$ (Figure 3). The solution to this first variation is accessible to 8th-grade students because it can be obtained by using the triangle similarity and the Pythagorean theorem.

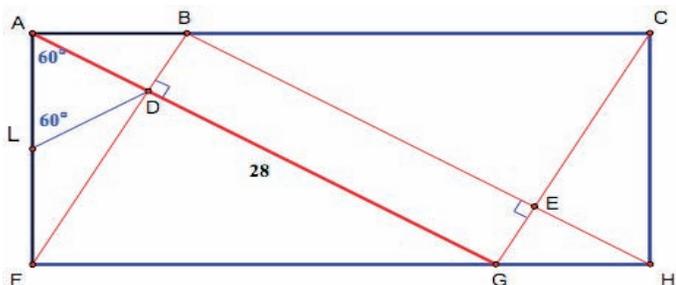


Figure 3. Assume that $\triangle BEGD$ is a rectangle.

First, notice that the auxiliary line LD shows that $\triangle ALD$ is an equilateral triangle and that $\triangle LFD$ is an isosceles triangle (M4G1). Since two parallel lines AC and FH are cut by transversals BF and CG, we can find the measures of all the inner angles (M8G1b) and observe that the triangles, except for $\triangle ALD$ and $\triangle LFD$, are the 30° - 60° - 90° right triangles (M4M2). Furthermore, since $BD = EG$ and $DG = BE$ and the ratios $AC/FH = AF/CH = BF/CG = 1$ (M7G3b and c), we know that $\triangle AGC \approx \triangle HBF$. In the same way, we know that $\triangle AFD \approx \triangle HCE$, $\triangle BDA \approx \triangle GEH$, and $\triangle DFG \approx \triangle ECH$ (M8G1d). It only remains to find the dimensions of $\triangle AFD$, $\triangle BDA$, and $\triangle DFG$.

If the variable x represents the length of AD (M5A1), then $AL = LD = LF = x$, and $AF = 2x$ because $\triangle ALD$ is equilateral and $\triangle LFD$ is isosceles. Since all the right triangles are similar (use the Angle-Angle-Angle condition of similarity, M7G3), we know in particular that $\triangle AFD \sim \triangle AGF$, and because corresponding sides of similar triangles are proportional, we can find $x = 7$. By applying the Pythagorean theorem to the right triangle $\triangle AFD$ (M8G2), we have $DF = \sqrt{147} = 7\sqrt{3}$ inches (M8N1). The fact that $\triangle ADF$ and $\triangle BDA$ are similar provides $BD = (7\sqrt{3})/3$ inches. The rest of the dimensions of $\triangle ADF$, $\triangle ADB$, and $\triangle DFG$ can be easily computed by using the Pythagorean theorem.

A slight variation of this problem (Figure 4) makes it appropriate for high school students because it can no longer be solved without using trigonometric ratios (MM2G1, MM2G2); in particular, we assume that $\angle FAD$ is 50° instead of 60° . Let $AD = x$ and $DG = y$. Then $x + y = 28$. Since $\triangle AFD$ is a right triangle and $\angle FAD = 50^\circ$, $DF = x \tan 50^\circ$. Since $\triangle FDG$ is also a right triangle and $\angle FDFG = 50^\circ$, $x(\tan 50^\circ)^2 = y$. By solving a linear system consisting of $x + y = 28$ and $x(\tan 50^\circ)^2 = y$ (see M8A5), we obtain $x = 28/((\tan 50^\circ)^2 + 1) \approx 11.57$ and $y \approx 16.43$. Since all angles were identified as 50° , 40° , or 90° , the rest of the dimensions can be found using trigonometric ratios.

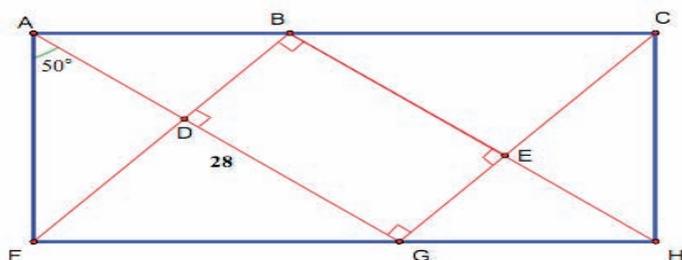


Figure 4. Assume that the angle "FAD is 50° ."

Another approach to solve the problem at the high school level is to use the Pythagorean theorem and solve quadratic equations for one variable (MM2A4). Let $AD = x$, then $DF = x \tan 50^\circ$. Using the Pythagorean theorem for the right triangle $\triangle AFD$, we have:

$$\begin{aligned} DF^2 + AD^2 &= AF^2 \\ (x \tan 50^\circ)^2 + x^2 &= AF^2 \\ [(\tan 50^\circ)^2 + 1] x^2 &= AF^2 \end{aligned}$$

By solving the quadratic equation using the Pythagorean

Theorem in the same way for $\triangle DFG$, we obtain the equation $[(\tan 50^\circ)^2 + 1]x^2 - 56x + 28^2 = FG^2$, and lastly, applying the

Pythagorean theorem to $\triangle AFG$ we obtain the equation $AF^2 + FG^2 = 28^2$. This quadratic equation can be solved for $x > 0$, and we find that $x = 28/((\tan 50^\circ)^2 + 1) \approx 11.57$. Therefore $AD \approx 11.57$ inches, $DG = 28 - x \approx 16.43$ inches, and DF , and BD can be found using the tangent of 50° . The lengths of AB , AF , and FG can then be found by using the Pythagorean theorem. Since $\square BDGE$ is a rectangle, $EG = BD$ and $BE = DG$ (MM1G1). The fact that $AC = FH$ and $\angle ABF = \angle ACG = 50^\circ$ implies $FB = GJ$ so $\square BFGC$ must be a parallelogram (MM1G3). Thus, $BC = FG$ and $EC = DF$. In a similar way, we have $GH = AB$ and $EH = AD$. The remaining measures and angles can be found by applying the properties of rectangles and similar triangles.

In addition to the content standards addressed, these tasks promote the Georgia Process Standards because in order to solve problems students must apply and adapt appropriate strategies, make mathematical conjectures, and develop arguments and proofs. Students need to understand how mathematical ideas connect in order to organize and consolidate their mathematical thinking, to use the language of mathematics to express their ideas in a coherent, clear, and precise way, and to demonstrate the validity of their reasoning.

We have made slight modifications to the assumptions of the Window Pain Task from the GPS Frameworks to create new problems at the 8th grade and high school level. As we have shown, these alternative assumptions changed not only the mathematical content of the problem but also sometimes made the problems too difficult to be solved using 8th grade mathematics. It is important to make explicit all the conditions of the mathematical problems posed in the classroom, not only because students should develop the mathematical habit of identifying what is known when beginning a problem, but also because mistaken assumptions may lead students to work on a version of the task that is beyond any reasonable expectation of their ability.

Making explicit the assumptions of a task and modifying them requires mathematical understanding and is a key component of pedagogical expertise. The decisions this work entails are an essential part of a math teacher's role, but thinking about assumptions is not just for teachers. Tenth graders might be interested in exploring the significance of the assumption that the angle $\angle FAD$ is 60° (Figure 4) and 8th graders might learn deep mathematics discussing the importance of the assumption that the angle $\angle ABD$ is 60° (Figure 1). One can begin these discussions by asking, What if? What if not?

Revisiting and manipulating the conditions and assumptions of a mathematical problem during a class discussion can lead students to another dimension of problem solving, called problem posing, which is a highly recommended activity (NCTM, 2000) that requires deep understanding of the mathematics involved (Brown & Walter, 2005; Kojima & Miwa, 2008; Polya, 1957; Silver et al, 1996). Expert teachers, according to research literature, tend to spend considerably more time than novice teachers formulating and reformulating problems (Silver & Marshall, 1989). The process of reformulating a problem leads to modifications in the level of task difficulty, differentiation of the task for the needs of a diverse population of students, or even the introduction of new mathematical ideas. The problems that we have presented here are a few examples of the many ways in which these concepts can be used to extend and enrich any mathematics task or problem.

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Reflections, an official publication of the Georgia Council of Teachers of Mathematics, serves as a resource to inform the membership of Council activities and provide resources to enhance the teaching and learning of mathematics in Georgia.

Making Mathematics FUN

Dr. Rock's Math Mystery

by David Rock
Columbus State University
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Elementary Brain Teaser

From Last Issue

Sneaky Series

Find the next two number in the pattern below:

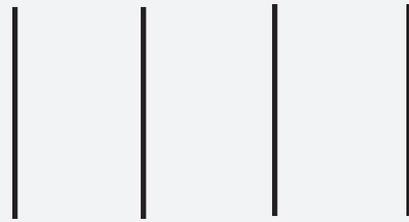
9, 18, 11, 16, 13, 14, 15, 12, 17, 10, ____, ____

Sneaky Series Solution: 19, 8.

Look at every other number, 9, 11, 13, 15, 17, and 18, 16, 14, 12, 10

New One!

A Perfect 10!



Observe the four straight, vertical line segments above.
Add 5 more straight line segments to make 10.

Challenge Round

From Last Issue

Four Digit Miracle

Find a positive four digit number with different digits such that: the difference of the four digit number with its digits arranged from greatest to least and arranged from least to greatest is equal to the original four digit number.

Four Digit Miracle Solution: 6174.

Because $7641 - 1467 = 6174$

New One!

That's That!

If $THAT = (AH)(HA)$ and each letter represents a unique digit, determine THAT.



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