I Can't Remember Which Fraction to Keep or Flip:
Building Understanding of Fraction Division with the CRA Instructional Model

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Teaching fraction division is challenging, especially when instruction focuses on teaching the procedure of keep-change-flip without a deep conceptual understanding of how and why that algorithm works. However, using the Concrete-Representation-Abstract (CRA) Instructional Model, evidence-based practice is a way to address this challenge. The CRA Instructional Model is helpful for students with and without disabilities when teaching any mathematical operation (Gersten et al., 2009; Siegler et al., 2010). Students begin with building concrete models using physical or virtual manipulatives, then transition to representational drawings, and finally, make connections to abstract computation-based strategies. Throughout each phase, teachers support students in making sense of the problem and connecting their models or computational strategies to the context of the mathematical word problem.

The CRA Instructional Model (see Figures 1 and 2) is a systematic way to build from conceptual understanding to procedural fluency. Students must begin in the concrete phase and then learn the subsequent phases for teachers to implement it with fidelity. You will notice the overlapping of each of the phases–the teacher must facilitate students making explicit connections between the adjacent phases. It is also essential to understand that using the CRA Instructional Model takes time. Teachers will not have students learn all phases in a single lesson or a single week. Students will be ready to move on to the next phase at different times, so teachers will need to meet the varied needs of students.

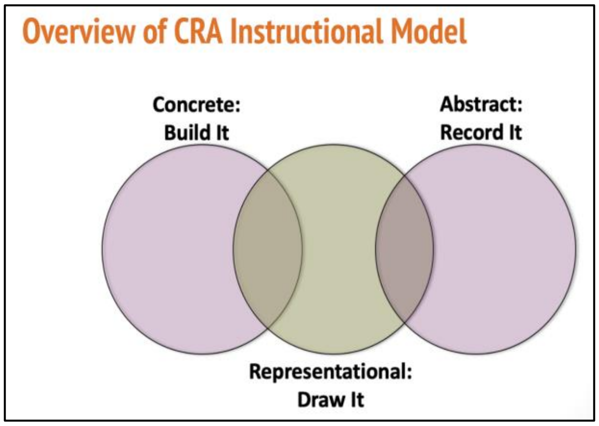

Figure 1: Overview of the CRA Model

Phase 1	Present the Context
Phase 2	Concrete
Phase 3	Overlap of Concrete and Representational
Phase 4	Representational
Phase 5	Overlap of Representational and Abstract
Phase 6	Abstract

Figure 2: Each Phase of the CRA Model

Content Standards Progression for Fraction Division

When building on prior knowledge, teachers should engage students in both sharing (how many in each group) and measurement (how many groups) division word problems with whole numbers and manipulatives. These activities will allow students to conceptually understand what each part of their model represents and demonstrate their use of manipulatives to solve it. It is essential for students to master the division of whole numbers before teachers introduce fraction division. When teachers introduce fraction division in 5th-grade, they should provide students with a word problem where a whole number is divided by a unit fraction ($6 \div \frac{1}{2}$) and where a unit fraction is divided by a whole number ($(\frac{1}{2}) \div 6$). Once students can divide unit fractions and whole numbers at the mastery level, they are ready to transition to any combination of dividing fractions by fractions, a 6th-grade standard. For example, we can give students tasks where a fraction less than one is divided by a unit fraction (e.g., $\frac{5}{6} \div \frac{1}{6}$ or $\frac{2}{3} \div \frac{1}{6}$) and then any fraction divided by any fraction (e.g., $\frac{5}{6} \div \frac{2}{3}$; $2\frac{3}{4} \div \frac{2}{3}$; $5\,\frac{2}{3} \div 1\frac{3}{4}$). As such, the task included in this paper is only appropriate for a 6th-grade class *4fter* sufficient time with tasks where a whole number is divided by a unit fraction, a unit fraction is divided by a whole number, and a fraction less than one is divided by a unit fraction. However, you can modify this task to be appropriate for 5th-grade students or early in the 6th-grade year. You can explore the Georgia K-8 Mathematics Standards (GaDOE, 2021) for fraction division, which include the expectations and evidence of student learning, at https://www.gadoe.org/Curriculum-Instruction-and-Assessment/Curriculum-and-Instruction/Documents/Mathematics/Georgia-K12-Mathematics-Standards/Georgia-K-8-Mathematics-Standards.pdf.

Standards for Mathematical Practices

Using the CRA Instructional Model provides students opportunities to engage in several Standards for Mathematical Practice. A focus on standard MP2: *Reason abstractly and quantitatively* encourages students to make sense of the context and relate it to their model, whether using concrete manipulatives, representational drawings, or abstract symbols. MP6: *Attend to precision* encourages students to correctly label each model or drawing, referring to the correct whole and being specific about the units as they answer the word problem. For example, if the question asks how many *pitchers*, they should not have the label of *gallons* in their answers. Lastly, as in all discourse-based classrooms, students should be comfortable engaging in MP3: *Construct viable arguments and critique the reasoning of others* as they work in small groups to make sense of what the word problem is asking, how they are modeling or representing the problem, and what information they are using to solve the problem. To explore descriptions of these and the other mathematical practices, you can refer to the Standards for Mathematical Practice (NGA Center & CCSSO, 2010, p. 6-8):

https://ccsso.org/sites/default/files/2017-12/ADA%20Compliant%20Math%20Standards.pdf

Mathematical Modeling

Teachers should incorporate mathematical modeling into every grade level by giving students opportunities to deepen their understanding of the content. The CRA Instructional Model (see Figures 1 and 2) can easily be integrated into Georgia's Mathematical Modeling Framework (see Figure 3) (GaDOE, 2021, p.116). As students "gather information, make assumptions, and define variables," they are engaging in Phase 1 of the CRA model (see Figure 2), where they are making sense of the context in the word problem. "Create a model" connects to Phases 2-4 as students work through the concrete to representational phases. Students continue to persevere through the word problems by analyzing, revising, and evaluating their models. They interpret solutions by connecting the context of the word problem to the concrete or representational model. Engaging in mathematical models lays the foundation for eventually transitioning into Phase 6, where students use abstract computation to solve word problems.

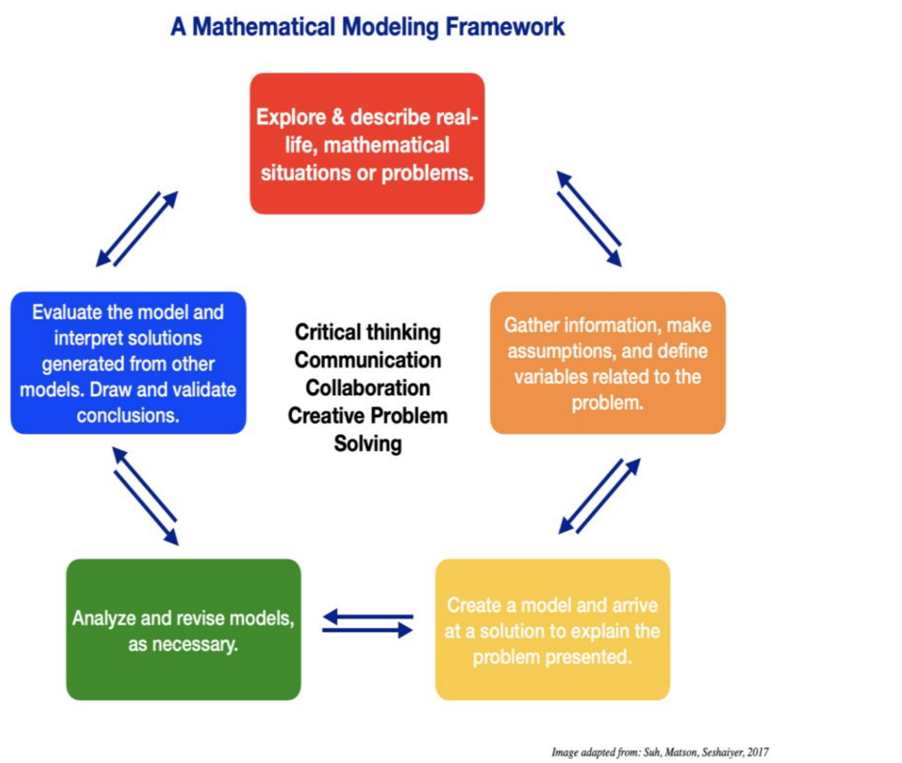

Figure 3: Mathematical Modeling Framework (GaDOE, 2021, p. 116)
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Effective Math Teaching Practices

While the previous standards and frameworks described above have focused on what students should know and be able to do, some standards focus on what teachers should do when planning, teaching, and assessing mathematics. The National Council of Teachers of Mathematics (NCTM) identified eight mathematics teaching practices that all teachers should use to maximize student learning (2014). Using the CRA Instructional Model provides authentic opportunities for teachers to use two of these teaching practices, "us[ing] and connect[ing] representations" and "building procedural fluency from conceptual understanding" in a meaningful way (NCTM, 2014, p. 10).

Applying the CRA Instructional Model with Fraction Division

The following outline of the CRA Instructional Model (see Figures 4 through 8) describes how students could engage in fraction division and examples of questions teachers can use to facilitate critical thinking (and potential students' responses) for a single problem across all CRA phases. We describe how moving through each phase creates the opportunity to effectively scaffold from developing a conceptual understanding using concrete models and then representations to develop procedural fluency.

Throughout the phases, the teacher facilitation questions are similar, and the repetition is purposeful. Students will move through the phases at varying paces. Often, it takes weeks to go through the entire CRA progression for a given concept. Therefore, it is necessary and appropriate to revisit questions to emphasize key concepts within each phase so that students understand the context of the word problem. For instance, we want students to continually refer back to the context of the word problem and connect it to their concrete models, representational drawings, and abstract computation strategies. Additionally, these questions encourage students to make connections to their drawings when they are in the abstract phase. The goal is for students to understand how they came up with each fraction, what it represents, and how it connects to an operation.

<table>
<tr><td colspan="2">Phase 1: Present the Context</td></tr>
<tr><td colspan="2">Example:
Abigail has $\frac{5}{6}$ of a gallon of punch. She wants to put the punch into pitchers that each hold $\frac{2}{3}$ of a gallon. Using all the punch, how many pitchers can Abigail fill?</td></tr>
<tr><td>Student Engagement:
• Students should read the problem a few times to become familiar with the context.
• Students should ask themselves as they read (1) What information do you know? (2) What are you trying to find?
• Students should engage in discourse with their peers to help them make sense of the problem.</td><td>Teacher Facilitation:*(possible responses in red)
• What is happening in the problem? Abigail is putting her leftover punch into pitchers.
• What whole is the $\frac{5}{6}$ referring to? One gallon
• What whole is the $\frac{2}{3}$ referring to? One gallon
• Which whole is the question referring to? One pitcher
• Do you have more or less than an entire pitcher of punch leftover? How do you know? More. One pitcher is $\frac{2}{3}$ of a gallon and we have $\frac{5}{6}$ of a gallon of punch leftover. We know $\frac{5}{6}$ is greater than $\frac{2}{3}$, so we have more punch leftover than the amount it takes to fill one pitcher.</td></tr>
</table>

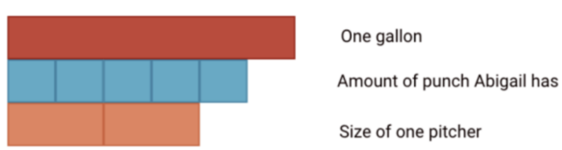
Figure 4: Phase 1 Example

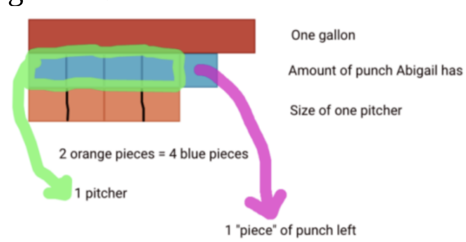
*We have included some possible questions and potential student responses to support your implementation of this task. This list is not exhaustive and we encourage you to develop additional questions based on your students' needs.

Describing the Mathematics of Phase 1:

When introducing the task, we need to make sure students understand the entire context of the word problem, not just search for keywords. Asking students questions encourages them to think critically to understand the context better, helps them make sense of how they might model or draw their strategy, and identify what each part of their model represents. Phase 1 is essential and should not be rushed. It is vital that the teacher facilitates students' critical thinking and does not make sense of the problem for their students. Phase 1 provides students opportunities to engage in productive struggle as they continue making sense of the task.

Teachers often use one strategy to support students in making sense of a context: to use realia. Having an actual pitcher available for students to see can help them connect to the word problem. However, teachers must be aware that when students begin modeling the problem using math tools, like fraction tiles, some might struggle to see how the model represents the physical pitcher. To support students, it is essential for teachers to continue to make explicit connections between the context and the concrete model (see the sample teacher facilitation questions in Figure 5).

<table>
<tr><td colspan="2">Phase 2: Concrete</td></tr>
<tr><td colspan="2">Example:*
Abigail has $\frac{5}{6}$ of a gallon of punch. She wants to put the punch into pitchers that each hold $\frac{2}{3}$ of a gallon. Using all the punch, how many pitchers can Abigail fill?




Students will likely exchange the two orange pieces for four teal pieces to show equivalence.

This model was created using virtual manipulatives in the BrainingCamp platform. To learn more about this resource, see https://www.brainingcamp.com/</td></tr>
<tr><td>Student Engagement:
● Use physical or virtual manipulatives to create a concrete model (*In this</td><td>Teacher Facilitation: (possible responses in red)
● What does the red rectangle represent? One gallon
● What do the five teal pieces represent? The amount of punch Abigail has leftover which is $\frac{5}{6}$ of a gallon
● What do the two orange pieces represent? The size of one</td></tr>
</table>

example, we are using virtual fraction tiles). ● Label the concrete model based on the context of the word problem. ● Use the concrete model to solve the problem.	pitcher, so $\frac{2}{3}$ of a gallon ● Do we have enough punch to fill one full pitcher? Why? Yes, because the length of the five $\frac{1}{6}$ teal pieces are longer than the length of the two $\frac{1}{3}$ orange pieces. ● How can we use what we know about equivalent fractions to make sense of the size of orange pieces? Why? How does that change our answers to the previous questions? Exchange each $\frac{1}{3}$ orange piece with two $\frac{1}{6}$ teal pieces, so all pieces are $\frac{1}{6}$ teal pieces, which means all pieces are the same size. Now that we have the same size pieces (sixths), we can see it takes four $\frac{1}{6}$ teal pieces to fill one pitcher, and we have five $\frac{1}{6}$ teal pieces of punch. So we can fill one full pitcher and have one more $\frac{1}{6}$ teal piece of punch leftover.

Figure 5. CRA Phase 2 Example

* This is just one example of how students might build a concrete model using virtual fraction tiles. To model this problem, students could use other concrete or virtual tools, like Cuisenaire rods, pattern blocks, fraction circles, paper folding, or a number line.

Describing the Mathematics of Phase 2:

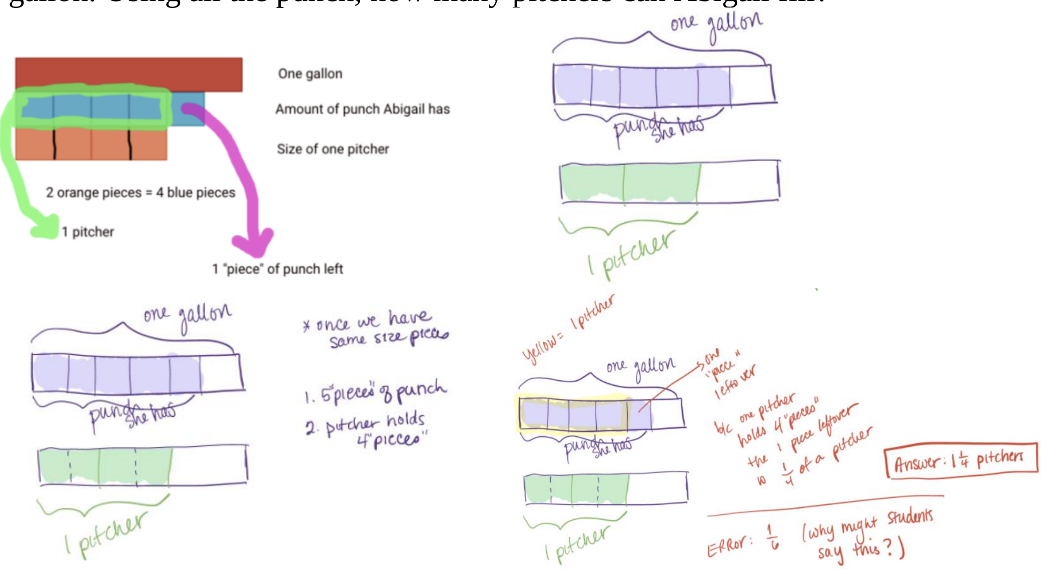
Students should stay in the concrete phase for some time as they model various fraction division word problems using physical or virtual models. They should be able to build and use those models to find and explain the solution before moving to the next phase of the CRA Instructional Model. Additionally, students should label each part of the model, as pictured in the example in Figure 5, where the red tile represents one gallon, the two $\frac{1}{3}$ orange tiles represent one pitcher, which is $\frac{2}{3}$ of a gallon, and the five $\frac{1}{6}$ teal pieces represent the amount of punch Abigail has, which is $\frac{5}{6}$ of a gallon. We want to make sure students do not label the red tile as "one whole" without connecting it to the context of the word problem (by asking "It is 'one whole' of *what*?") because the *whole* could change throughout the word problem. For example, in the context of Abigail's problem, one whole pitcher and one whole gallon are two different representations of a whole, but they do not equal the same amount.

As students begin to solve using their concrete models, they realize they can trade each $\frac{1}{3}$ orange piece for two $\frac{1}{6}$ teal pieces using their background knowledge of equivalent fractions. Once they do this, it allows students to notice that the five $\frac{1}{6}$ teal pieces of punch that Abigail has leftover and four $\frac{1}{6}$ teal pieces that makeup one pitcher are all the same size. With teacher facilitation through open-ended questions, students should conclude that they can fill one whole pitcher and have one piece leftover. To make sense of the leftover piece, students should compare the leftover piece to the total number of pieces that will fill one pitcher. They will eventually realize the leftover piece is $\frac{1}{4}$ of the pitcher. To facilitate this realization, you may need to ask students to reflect on how they labeled each model.

A common misconception that will likely arise is that students may compare the one leftover piece to the red representation of a gallon, which will result in identifying this piece as $\frac{1}{6}$ of a gallon. If this happens, ask students to look back at the context of the word problem, and have the students state which whole the question is referring to. This should help students realize they need to state how many *pitchers* Abigail needs, which requires them to compare it to four pieces (the number of pieces that make up one pitcher). The comparisons made among the whole and parts of the labeled concrete models help students conclude the answer is $1\frac{1}{4}$ pitchers that can be filled with the total amount of punch that Abigail has.

You might have some students say that the answer is 2 pitchers instead of $1\frac{1}{4}$ pitchers. While it is true that Abigail would need 2 pitchers to hold all of the punch, the question is asking how many pitchers Abigail can *fill* using all of the punch. She can fill 1 whole pitcher and $\frac{1}{4}$ of another pitcher; she cannot fill 2 full pitchers. Engaging in a discussion around this situation will engage students in MP6 *Attending to precision* as well as MP3 *Constructing viable arguments and critiquing the reasoning of others*.

Once students have mastery of solving fraction division word problems using the concrete models (Phase 2), they are ready to move on to Phase 3.

<table>
<tr><td colspan="2">Phase 3: Overlap of Concrete and Representational</td></tr>
<tr><td colspan="2">Example:
Abigail has $\frac{5}{6}$ of a gallon of punch. She wants to put the punch into pitchers that each hold $\frac{2}{3}$ of a gallon. Using all the punch, how many pitchers can Abigail fill?

</td></tr>
<tr><td>Student Engagement:
● Build the concrete model and simultaneously draw a representation of the concrete model.
● Label the concrete model and drawing based on the context of the word problem.
● Make connections</td><td>Teacher Facilitation: (possible responses in red)
● How does your drawing connect to the model? Both represent the amounts given in the word problem context; see Describing the Mathematics of Phase 2 for these details.
● How are they alike? The concrete model and drawing show the entire gallon, breaking each of the two $\frac{1}{3}$ pieces from the pitcher into four $\frac{1}{6}$ pieces to get the same size pieces and label each part of the model and drawing.</td></tr>
</table>

between how the concrete model and the drawing are used to solve the problem.	• How are they different? In the drawing, we needed to show the whole gallon each time we drew the pitcher or the amount of punch (shaded part of the whole rectangle/gallon), but in the concrete model, we only had to model the amount given (five $\frac{1}{6}$ teal pieces to represent the amount of punch Abigail has which is $\frac{5}{6}$ of a gallon, or two $\frac{1}{3}$ orange pieces for the size of the pitcher which is $\frac{2}{3}$ of a gallon) • How does your drawing show where you created equivalent fractions? How does that connect back to the word problem? When we broke each of the two $\frac{1}{3}$ green shaded pieces into two pieces; it created the equivalent fraction of four $\frac{1}{6}$ pieces; this shows the purple and green pieces are now the same size. So the pitcher is $\frac{2}{3}$ of a gallon, which is equivalent to $\frac{4}{6}$ of a gallon. • Why do you have to show the whole in every drawing? To show how many pieces the whole gallon is being broken into, which helps us identify the size of each piece.

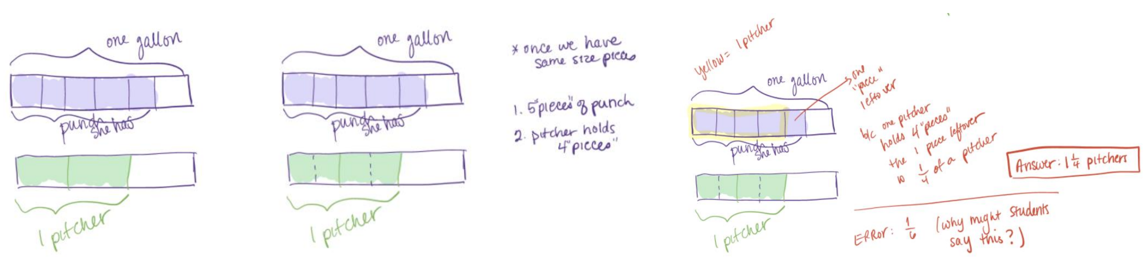
Figure 6. CRA Phase 3 Example

Describing the Mathematics of Phase 3:

This phase aims for students to begin recording the concrete model they build as a drawing. Students must build and record simultaneously to make connections between their concrete models and representational drawings. Teachers need to help make these connections explicit by facilitating student discourse.

Notice in the illustration above (see Figure 6) that just as we expect students to label their concrete models, we also expect them to label their representations (drawings) based on the context of the word problem. Remember, teachers need to support their students in making explicit connections between the concrete model and the representation (drawing) as follows: The representation (drawing) shows one whole rectangle, which is the same as the red piece in the concrete model; both represent one whole gallon. The representation (drawing) shows $\frac{5}{6}$ of the rectangle shaded in purple, which is the same as the five teal pieces in the concrete model; both represent the amount of punch Abigail has, which is $\frac{5}{6}$ of a gallon. The representation (drawing) shows $\frac{2}{3}$ of the rectangle shaded in green, which is the same as the two orange pieces in the concrete model; both represent the size of one pitcher which is $\frac{2}{3}$ of a gallon.

Students must stay in this overlap phase until they start to ask: *Do I have to keep building the model? Can't I just draw it?* These questions will indicate that the student no longer needs the concrete model and can draw the representations. Keep in mind that students will be ready to transition to the representational phase at different times. Students should be allowed to use physical or virtual manipulatives to support their learning until they are ready to move to the representational phase.

<table>
<tr><td colspan="2">Phase 4: Representational</td></tr>
<tr><td colspan="2">Example:
Abigail has $\frac{5}{6}$ of a gallon of punch. She wants to put the punch into pitchers that each hold $\frac{2}{3}$ of a gallon. Using all the punch, how many pitchers can Abigail fill?


</td></tr>
<tr><td>Student Engagement:
● Create a drawing to represent the word problem.
● Label the drawing based on the context of the word problem.</td><td>Teacher Facilitation: (possible responses in red)
● What does the whole rectangle represent? One gallon
● What do the five purple pieces represent? The total amount of punch Abigail has is $\frac{5}{6}$ of a gallon
● What do the two green pieces represent? The size of one pitcher, which is $\frac{2}{3}$ of a gallon</td></tr>
</table>

• Use the drawing to solve the problem.	• Do we have enough punch to fill one full pitcher? Why? Yes, because the purple area model (the amount of punch Abigail has) is longer than the green area model (the size of the pitcher)

Figure 7. CRA Phase 4 Example

Describing the Mathematics of Phase 4:

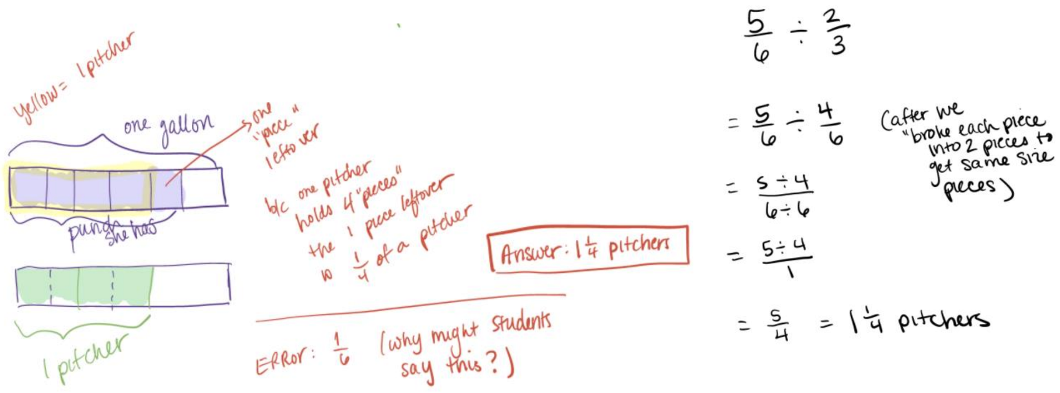
During the representational phase, we want students to continue to label each drawing. One way students might represent the word problem in a drawing is to create an area model to represent the punch Abigail has and another area model to represent the size of the pitcher (see Figure 7). After students draw these area models, they can use their knowledge of equivalent fractions to break the two $\frac{1}{3}$ pieces representing the size of the pitcher into four $\frac{1}{6}$ pieces. They should notice that the four $\frac{1}{6}$ pieces in the area model representing the size of the pitcher are the same size as the five $\frac{1}{6}$ pieces in the area model representing the punch Abigail has.

Through further questioning, students will come to understand that it takes four $\frac{1}{6}$ pieces to fill one pitcher, so we have enough pieces to fill one entire pitcher and have one piece leftover. Students must then make sense of the size of the leftover piece. Is it $\frac{1}{6}$ or $\frac{1}{4}$? Again, we need to make sure students understand which whole we are referring to (see the discussion in Describing the Mathematics of Phase 2 to revisit how to support your students in making sense of this and address common misconceptions). The comparisons made among the whole and parts of the labeled drawing help students conclude the answer is $1\frac{1}{4}$ pitchers that can be filled with the total amount of punch that Abigail has.

Once students have mastery of solving fraction division word problems using the representational drawings (Phase 4), they are ready to move on to Phase 5.

Phase 5: Overlap of Representational and Abstract

Example:
Abigail has $\frac{5}{6}$ of a gallon of punch. She wants to put the punch into pitchers that each hold $\frac{2}{3}$ of a gallon. Using all the punch, how many pitchers can Abigail fill?



Student Engagement:

- Simultaneously draw the representation and record the abstract computation.
- Label the drawing based on the context of the word problem and identify how the context is represented in the abstract computation.
- Make connections between how the drawing and abstract computation is used to solve the problem.

Teacher Facilitation: (possible responses in red)

- How does each step of computation connect to the drawing? See Describing the Mathematics of Phase 5 for these details.
- Why did we find an equivalent fraction for $\frac{2}{3}$? We need the same sized pieces to compare the amount of punch Abigail has with the pitcher.
- Where is that shown in the drawing? When you partition the two $\frac{1}{3}$ green shaded pieces into four $\frac{1}{6}$ green shaded pieces
  Symbolically? When we rewrote $\frac{2}{3}$ as the equivalent fraction of $\frac{4}{6}$ from steps 1 to 2
- Where is the whole "renamed" in the drawing? When you use the pitcher as the whole to reason about the size of the leftover piece of punch
  Symbolically? From steps 2 to 3, when you shift from sixths to fourths.
- How does simplifying the denominator connect to the drawing? It is the same as where the whole is "renamed" in the drawing (see above)
- Why is the quotient in fourths, not sixths or thirds? Where can you see that in the drawing? in the computation? Once we find the exact size pieces (equivalent fractions), we have four parts that the pitcher holds (which is what the question is asking us to find), so our answer is in terms of fourths. In

	the drawing it shows four equal pieces; in the computation, it is when we divide 5 by 4.

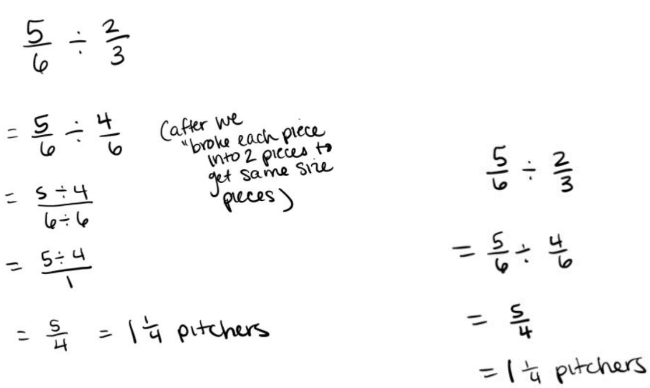
Figure 8: Phase 5 Example

Describing the Mathematics of Phase 5:

This phase aims for students to begin symbolically recording the abstract computation as they create the representational drawing. It is essential that students draw and record simultaneously and make connections between their representational drawings and abstract computation. Teachers need to help make these connections explicit by facilitating student discourse.

As students record the abstract computation, they should be able to explain how each fraction connects back to the context of the word problem. Remember, teachers need to support their students in making explicit connections between the representation (drawing) and the abstract computation (symbolic) as follows: The purple shaded area model represents $\frac{5}{6}$ of a gallon, which is the amount of punch Abigail has, and the green shaded area model represents $\frac{2}{3}$ of a gallon, which is the size of one pitcher. In step 2 of Figure 8, the drawing helps students understand the need to find the equivalent fraction of $\frac{2}{3}$ by breaking each piece into two $\frac{1}{6}$ pieces to create the equivalent fraction of $\frac{4}{6}$. This helps students make sense of the reason behind finding common denominators in the abstract computation because they need to find the same size pieces. Moving from step 2 to step 3 requires students to "rename" the whole. In step 2, both fractions were in sixths because students were initially comparing the amount of punch and the size of the pitcher to the one gallon. However, now students need to answer, "Using all the punch, how many *pitchers* can Abigail fill?" This question requires students to consider a new whole, the *pitcher*. Because the one whole pitcher comprises four equal-sized pieces, our new whole consists of fourths, that is why students record 5 ÷ 4 in step 4; our question is now asking: How many wholes (4 pieces) are in the given amount (5 pieces)?

Students must stay in this overlap phase until they start to ask: *Do I have to keep drawing the model? Can't I just solve it?* These questions will indicate that the student no longer needs the representational drawing and can solve the problem abstractly. Keep in mind that students will be ready to transition to the abstract phase at different times. Students should be allowed to use drawings to support their learning until they are ready to move to the abstract phase.

<table>
<tr><td colspan="2">Phase 6: Abstract</td></tr>
<tr><td colspan="2">Example: Abigail has $\frac{5}{6}$ of a gallon of punch. She wants to put the punch into pitchers that each hold $\frac{2}{3}$ of a gallon. Using all the punch, how many pitchers can Abigail fill?

</td></tr>
<tr><td>Student Engagement:
Solve the problem abstractly, using the fractional notation in the computation.
Identify each part of the abstract strategy based on the context of the word problem.
Explain why the abstract computation works</td><td>Teacher Facilitation: (possible responses in red)
Why do you rename $\frac{2}{3}$ as the equivalent fraction $\frac{4}{6}$? We need all our pieces/parts equal in size ($\frac{1}{6}$ of a gallon) to compare how much punch Abigail has compared to the size of the pitcher.
How does simplifying the expression to 5/4 connect to the context of the problem? The idea of reunitizing? When we create the same size pieces using equivalent fractions, we have five parts of punch, but four parts of punch can be used to fill one pitcher; this is where the whole is renamed/reunitized from one gallon to one pitcher.</td></tr>
</table>

mathematically.	• Why is the quotient fourths? Our one whole (the pitcher) holds four parts • Will this strategy work for any fraction division problem? How do you know? Yes, we can always find equivalent fractions, which helps us determine how many parts the new whole that we are referring to contains; and fraction division will always require renaming/reunitizing the whole

Figure 9. Phase 6 Example

Describing the Mathematics of Phase 6:

After students transition to the Abstract phase, it is essential to continue to connect the steps in the computation back to the context of the problem. We do not want fraction division to be about symbols; attaching meaning to those symbols is key to helping students build their procedural fluency from conceptual understanding.

When dividing fractions using abstract computation, students must find common denominators. This will help them understand how and why a new whole is created, also called *reunitizing*. Students created physical or visual common denominators in the concrete and representation phases by finding "same size pieces," but they compute it in the abstract phase when they rename $\frac{2}{3}$ as $\frac{4}{6}$. Next, teachers question students to consider the problem, by asking: *How many times does* $\frac{4}{6}$ *fit into* $\frac{5}{6}$*?* Or, within the context, by asking: *How many pitchers that are 4/6 of a gallon will be filled with* $\frac{5}{6}$ *gallons of punch?* We can see how fraction division is a measurement model where students find out how many groups of the divisor make up the dividend. Computationally, as they divide $\frac{5}{6}$ by $\frac{4}{6}$, they reunitize or create a new whole. Because the fractions are in sixths, teachers should ask: How many wholes (4 parts) are in the given amount (5 parts) or 5 ÷ 4? Once students interpret fractions as division, they will find that $\frac{5}{4}$ or $1\frac{1}{4}$ pitchers will be filled with Abigail's punch.

This alternative algorithm is an appropriate strategy for students to use when first entering the abstract phase. Students are not ready for the standard algorithm of keep-change-flip, although they may notice patterns that lead to the algorithm. If that happens, teachers can note the student's conjecture and gather evidence if this is *sometimes* or *always* true. This practice supports inquiry-based learning, where students explore these ideas as a class. While students may discover the standard algorithm of keep-change-flip, they must have a deep conceptual understanding of complex fractions before they can make sense of the mathematics behind the standard algorithm and truly understand *how and why* it works, which is the dual goal of procedural fluency.

Conclusion

The CRA Instructional Model supports students engaging in the Standards for Mathematical Practice and mathematical modeling. Moreover, it aligns with NCTM's Mathematics Teaching Practices, and it is an evidence-based practice that supports the learning of all students. We described a practical example of encouraging students to build an understanding of fraction division through the CRA phases using a word problem. This description included possible misconceptions that students may make and some tips for precise language to use when students are modeling, explaining, and justifying. We also provided possible questions to help students use their critical thinking skills, explore, and apply the CRA Instructional Model in each phase. Implementing evidence-based practices like CRA will encourage all students to understand mathematics deeply.

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