# Building Proof Bridges Interactively with Desmos

## Introduction

We experimented with using online tools in new ways to reconstruct face-to-face mathematics tasks into online collaborations during the time of COVID-19. We plan to build on what we learned during the pandemic to bring new student tools to our face-to-face classes.
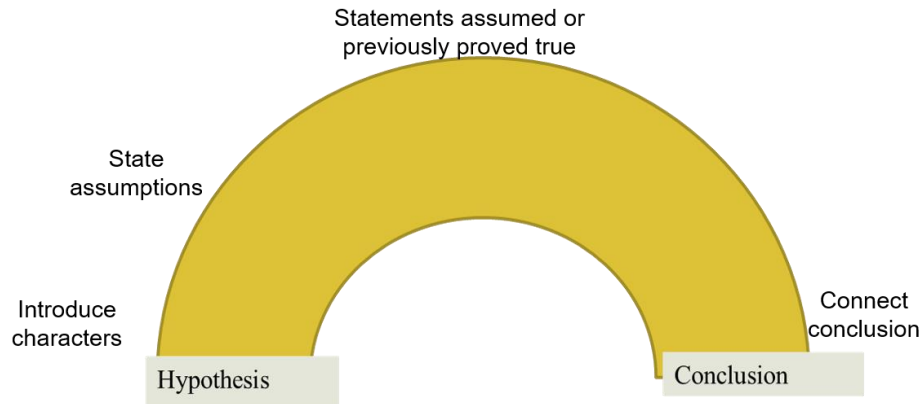
One new tool that we plan to continue using is Desmos, a free online graphing calculator and activity builder. It was easy to begin using it right away at a basic level for any topic. As we used it, we found that community resources helped us develop more sophisticated use. We share an example of using a community resource to translate a set of face-to-face proof-writing tasks into a Desmos activity set.

## Proof-writing in Face-to-Face Classes

Proof-writing, a creative and abstract problem-solving process, can feel distressing to students. Commonly acknowledged from lower grades to university courses is the difficulty in transitioning from seeing mathematics as mostly computational to seeing it as reliant on logic, reasoning, and writing skills. The Georgia Standards of Excellence addresses this difficulty by integrating standards of mathematical practice throughout the K-12 curriculum. At the university level, transitional courses are often provided to support students as they move from (often) procedural and computational courses to more abstract proof-writing courses. To support this process, Ha provided a visual framework (see Figure 1) for her students to use to know how to start, how to make progress, and how to know when a proof is done.

**Figure 1**

*Proof Writing Visual Framework*

Statements assumed or previously proved true

State assumptions

Introduce characters

Connect conclusion

Hypothesis

Conclusion

Writing a formal mathematical proof is a process of (1) starting with a given set of statements assumed or accepted as true, (2) creating a logical sequence of statements (e.g., definitions, previously proved results), and then (3) arriving at the desired conclusion. Embedded within this seemingly straightforward process is the messiness of brainstorming, trying a logical sequence that fails, making connections between consequences of failed attempts and the goals, and revising the sequence based on new knowledge.

Without opportunities to see the scratch work behind the scenes, students may feel anxiety when engaging in their own trying and failing. Additionally, proof-writing is an abstract process that is rarely taught concretely even though mathematics educators agree that generally, students do better with opportunities to start with concrete concepts and then move to abstract (e.g., Gravemeijer, 1997).

Ha noticed that, despite the visual framework as a support, her students still struggled with the abstract nature of proof-writing. She turned to using a *proof bridges* task set inspired by Selden et al. (2018) and Malagon (2019). The tasks push students to view proof-writing as a construction process where definitions and theorems are raw materials that can be handled, moved around, and revised. Individual, focused statements are written on cards. The cards make the proof-writing process more concrete so students can begin to abstract the hard work of proof-

writing. By abstracting, we mean that students can move from concrete to abstract by developing

internal strategies for determining which statements are relevant, visualizing different sequences

of statements, and noticing the "a-ha!" moment when a sequence reaches the desired conclusion.

In previous face-to-face classes, Ha implemented proof bridges tasks using physical

whiteboards or notecards (see Figure 2). Students used whiteboards or cards. They brainstormed

ideas for a proof, writing one sentence per board/card.

**Figure 2**

*Face-to-Face Collaboration in Proof Bridges Task*
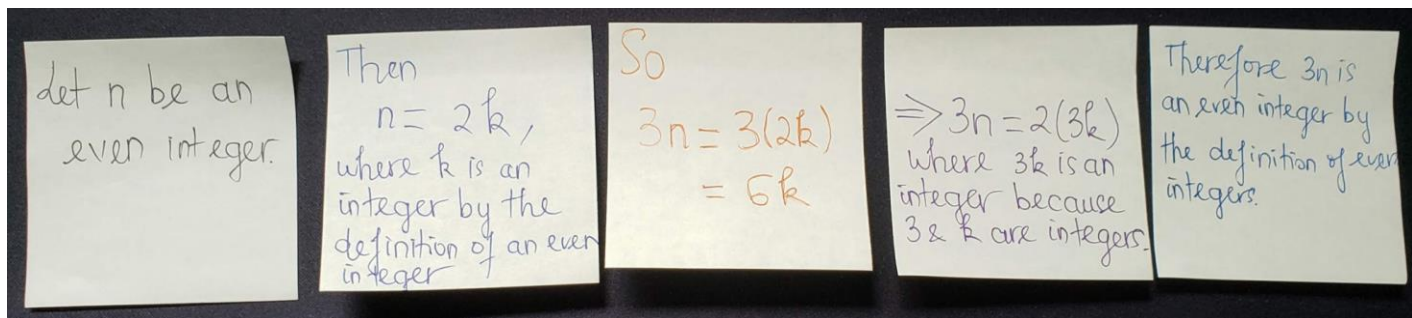


They stated the *given* information on one side and the *goals* information (the desired

conclusion) on the other. They then arranged and rearranged the brainstormed boards/cards to

build a path between the two sides, working from one side first and then the other. At times, they

removed a board or edited it as they refined their ideas and better visualized the path. In Spring

2020, as Ha moved her class online, she looked for a tool that would allow her to move the task

online and found Desmos Activity Builder.

**Overview of Desmos**

Desmos, launched by Yale student Eli Luberoff in 2011 as a free online graphing calculator to

support one-on-one tutoring, provides professionally constructed classroom activities and a

platform for individual teachers to customize existing activities or create entirely new activities

(Desmos, 2020c; Seaton, 2020). Their business model is to offer products free for teachers and

students while integrating the engine into profitable curricula and state assessments (Desmos, 2020c).

Because Desmos is based on a graphing calculator core, it is useful for mathematics tasks that incorporate tables, graphs, or calculations. It also supports importing images, videos, or graphs. The tool allows teachers to create, copy, and edit activities. A variety of student inputs allows teachers to quickly scan, view, and present student strategies and responses to support classroom discourse.

To support our translation of the in-class proof bridges task, we used the graphs feature of Desmos to allow students to experiment with sequences of provided statements and to provide students with immediate feedback when they created the correct sequence.

**Desmos Tasks: Solve Proofs "Puzzles"**

By searching for previously created Desmos activities, Ha found York (2016). York had shared the activity online in 2016 by Tweeting, "Took me forever, but worth it! Check out this self-checking @Desmos activity: Introduction to Geometry Proofs http://bit.ly/2e0NEWY #MtBoS." (MtBoS refers to the _math-twitter blogosphere_.) In the task, York provided a two-column proof background image with four movable statements indicated with a grabbable colorful point (e.g., "Division Property of Equality") for students to fill in the empty spaces (see Figure 3).

**Figure 3**

_Two-Column Proof Example with Draggable Statements (York, 2016)_

**Given:** $15y + 7 = 12 - 20y$

**Prove:** $y = \frac{1}{7}$

| | |
|---|---|
| 1) $15y + 7 = 12 - 20y$ | 1) |
| 2) $35y + 7 = 12$ | 2) |
| 3) $35y = 5$ | 3) |
| 4) $y = \frac{1}{7}$ | 4) |

Given ● 

Division Property of Equality ●

Addition Property of Equality ●

Subtraction Property of Equality ●

We first describe our revision of York's (2016) Desmos Activity, integrating the proof bridges metaphor. We then share a "how-to" guide for setting up the checkable graph.
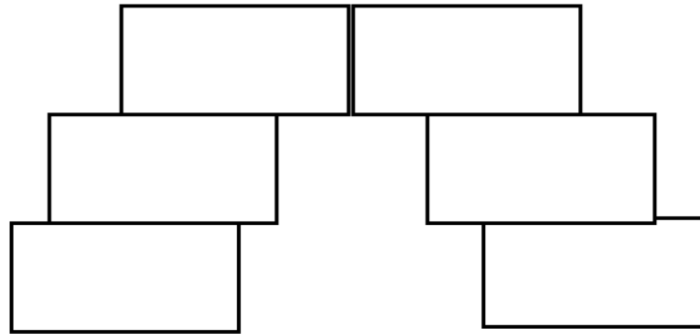
**Brief Description of Task**

We revised York's (2016) Desmos Activity by integrating the visual framework (Figure 1) with the individual statements (Figure 2). The task scaffolds proof construction by providing statements that students drag and drop into place. The task results in a paragraph proof rather than a two-column proof, so students focus on the logical ordering of statements. In the background, a logic statement checks the proof by comparing preset horizontal and vertical intervals to the location of the draggable statements. When all locations are within the preset intervals, a "Great job!" image pops up. Students may work on the task individually or collaboratively using video conferencing software and sharing their screens. Each student completes their task.

**Figure 4.**

*Desmos Proof Bridges Task*

Prove: If $n$ is an even integer, then $3n$ is an even integer.



Build the bridge by starting with the hypothesis (build on assumptions you start with) & conclusion (build backwards from where you need to end up).

Hypothesis (givens)

Conclusion (goals)

Let $n$ be an even integer.

Therefore $3n$ is an integer by definition of even integers.

Then $n = 2k$, where $k$ is an integer, by definition of even integers.

$3k$ is an integer because 3 & $k$ are both integers.

$3n = 3(2k)$ by substitution.

$3n = 2(3k)$ by commutativity of multiplication.

**How-To Guide for Creating Your Own Logic Puzzle**

The process of copying and editing a Desmos Activity can be as straightforward or complex as you choose. At a very basic level, you can simply copy and paste individual screens into your task. At the most complex, you can start from scratch and build your proof and images. You should choose what level makes sense for your comfort and needs. First, to create a Desmos Activity and familiarize yourself with basic functions use Desmos help pages (e.g., Desmos, 2020a). Then, navigate to https://bit.ly/3sNmGFf and decide whether to copy the whole activity or individual screens (e.g., Desmos, 2020b; 2021).

If you want to integrate your proofs, you can replace the images on our screens. If you simply want to build any graph with draggable images and instant feedback, then we describe York's (2016) strategy.

**Replacing Proof Statements.** If a proof is six or seven steps, then it is easy to replace the current images with your images. Currently, there are four basic folders: Background Stuff,

Draggable Points, Locations, and Images. Open the Images folder and click the "x" in the upper right corner of each statement image that you want to delete. To insert your images (created using your favorite strategy and software), open the Images folder on Desmos and click in any editable field of a file already in the Images folder to insert the new files automatically to the Images folder. It may be helpful to name them according to their order in the proof.

Select the files on your computer and drag-and-drop them to the menu where the Images folder is located. Resize the images so they will fit in the proof bridge and move them to a location beneath the bridge (e.g., Desmos, 2020d). Then connect each to a draggable point: (1) select the "Center" field below the image name, (2) delete the "(0,0)", and (3) type "(a1, b1)" for the first statement, "(a2,b2)" for the second statement, and so on. Check everything by clicking "Student Preview" and dragging the statements to their intended location. When you have them in order on the bridge, a "Great job!" image should appear.

**Creating a Graph with Instant Feedback.** Copy and paste a proof screen. Select the background "proof bridges" image and click "Delete" on your keyboard. Similarly, click and delete proof statements. Create a new proof bridge image using your favorite strategy and software, then drag the file from a computer folder into the Desmos "Background Stuff" folder. Resize the image and move it to a central location with space below for the statements (e.g., Desmos, 2020d). Create, insert, and prepare your statements following instructions in the previous section.

The next step is to tell Desmos where the statements (or any draggable images) should be for a correct answer. The logic is based on altering the height of the "Great job!" image. As long as one statement is out of place, its height is 0. As soon as all statements are located within the specified intervals, the height will increase, and the student will see "Great job!" pop up. Using

intervals rather than a set location is important because students might drag an image anywhere within a particular box.

Creating the intervals is somewhat tedious but fairly straightforward. It helps to be systematic, writing out intervals for the first statement, and then the second statement, and so on. For each statement, capture the $(x, y)$ values when the image is in the bottom left corner and again when it is in the upper right corner. For the example in Figure 5, the intervals will be {-5.6<a_1<-4.1}{-3.13<b_1<-0.92}.
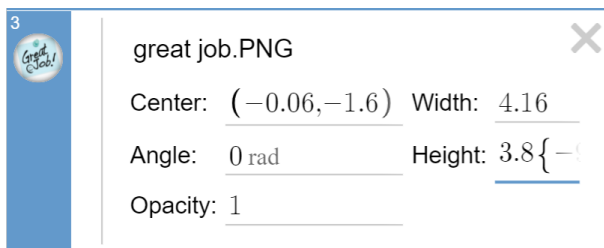
**Figure 5**

*Setting Intervals for Instant Feedback*
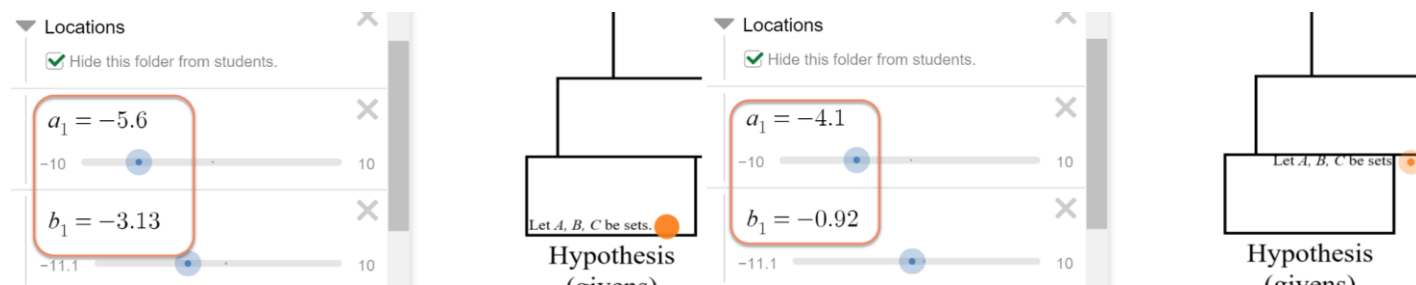


As you complete each interval, type it into the "Great Job!" image's editable Height field (see Figure 6).

**Figure 6**

*Great Job!*

Eryn prefers to type the intervals somewhere else and copy and paste them into the Height field. Unfortunately, although it is possible, the field does not accept pasted text unless it is formatted correctly. Hence, it may be easier to type directly into the field.

To copy and paste, first, click in the field, select all text, and copy all text. When you paste the text somewhere to edit it, it will look confusing and messy. The complexity is because it uses LaTeX formatting. For example, the interval *{-5.6<a_1<-4.1}* would become *\left\{-5.6<a_{1}<-4.1\right\}*. If you already feel comfortable with LaTeX formatting or if you are willing to experiment a bit, then eventually it will feel less confusing.

Continue to find the location intervals and type them into the Height field. When you are finished, click on "Student Preview" and check that everything is working.

## Lessons Learned

As we coped with teaching courses in person and online, we experimented with different presentations of content. Finding a way to translate this set of tasks---giving slips of paper to students to reorganize into a proof and then giving them opportunities to collaboratively build proofs using notecards or whiteboards---into a digital format was difficult. Building on the work of other teachers using Desmos Activity Builder made the transition more manageable.

Limitations of the initial task (where students re-organize someone else's reasoning using slips of paper or Desmos Activities) include that students must make sense of another person's reasoning and are unable to construct their statements. To balance these limitations, we follow up with tasks in a collaborative space (e.g., notecards or Google Jamboard). Benefits of using

Desmos Activities include that students can practice connecting and sequencing statements, working from both directions as needed. Additionally, a teacher can easily see student work on her screen and easily select the order to support class discussion.

## References

Desmos. (2020a). *Activity Builder: Overview.* https://support.desmos.com/hc/en-us/articles/207656923-Activity-Builder-Overview

Desmos. (2020b). *Duplicating and Editing Existing Activities.* https://support.desmos.com/hc/en-us/articles/207656973-Duplicating-and-Editing-Existing-Activities

Desmos. (2020c). *What we do.* https://www.desmos.com/about

Desmos. (2020d). *Add Image to Graph.*https://support.desmos.com/hc/en-us/articles/203291245-Add-Image-to-Graph#:~:text=To%20add%20an%20image%20to,d%20like%20to%20place%20it.

Desmos. (2021). *Copy and Paste Screens into an Activity.* https://support.desmos.com/hc/en-us/articles/360040778232-Copy-and-Paste-Screens-into-an-Activity

Gravemeijer, K. P. E. (1997). Mediating between concrete and abstract. In *Learning and teaching mathematics. An international perspective* (pp. 315-345). Psychology Press Ltd.

Malagon, A. (2019, July 31 - August 3). *Proof Bridges.* [Conference presentation abstract]. MAA MathFest 2019, Cincinnati, OH, United States. https://www.maa.org/sites/default/files/pdf/mathfest/2019/WebAbstracts.pdf

Seaton, J. (2020). *Eli Luberoff – CEO and Founder of Desmos.* The University of British Columbia, Vancouver Campus. https://virtual.educ.ubc.ca/wp/etec522/2020/02/01/eli-luberoff-ceo-and-founder-of-desmos/

Selden, A., Selden, J, & Benkhalti, A. (2018). Proof Frameworks: A Way to Get Started, *PRIMUS*, *28*(1), 31-45, DOI: 10.1080/10511970.2017.1355858

York, Sarah. (2016). *Introduction to Geometry Proofs.* Desmos Activity Builder. https://teacher.desmos.com/activitybuilder/custom/58012626a1924c7c08b7d94d