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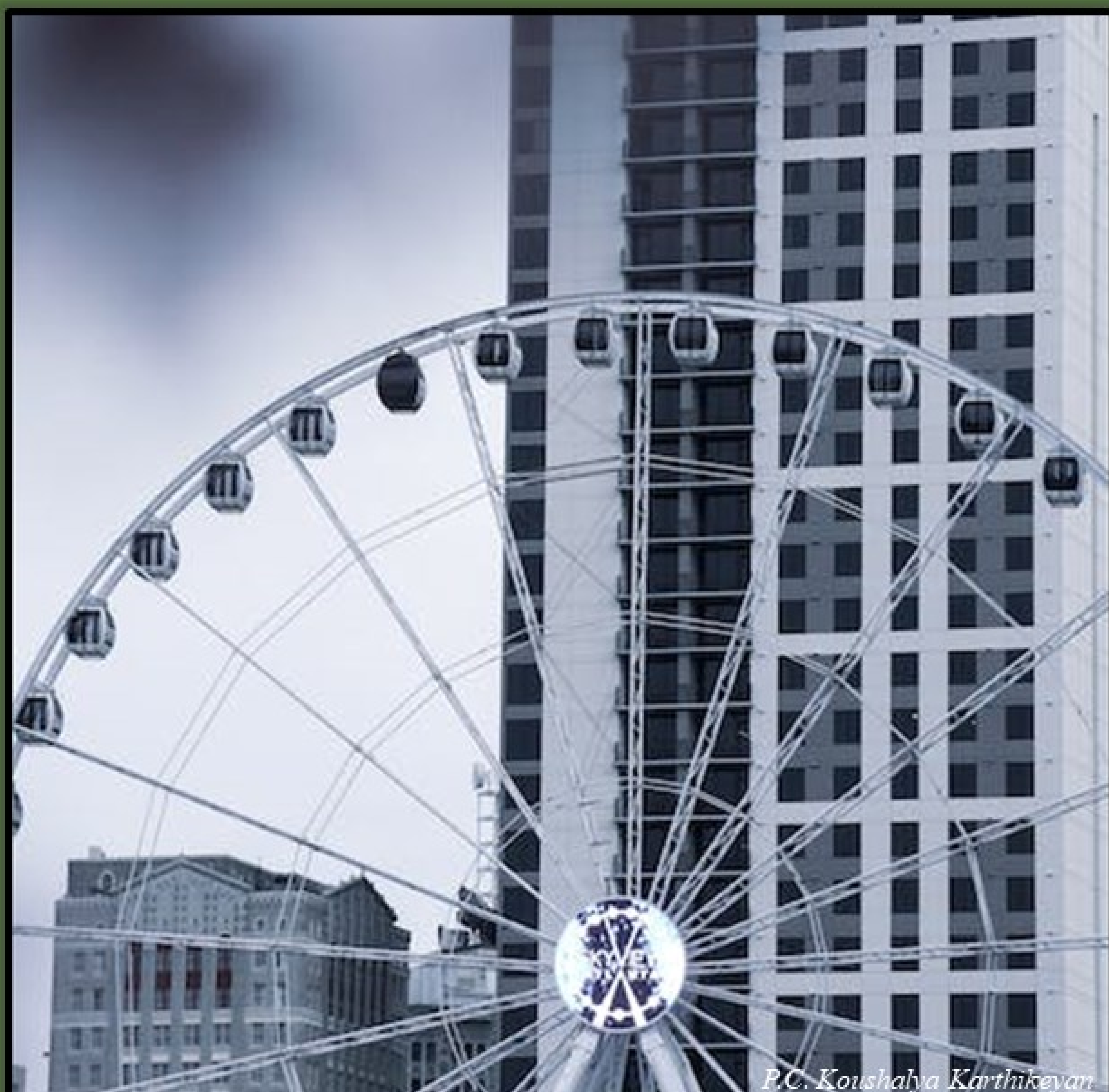
Reflections



Winter 2023 Vol XV, No 2

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Dear Reader,

2023 has offered a lot of new beginnings for our community. The Georgia Mathematics Standards rolled out at the beginning of this school year, and as professionals, we all jumped in head-first to adapt, learn our craft better, and collaborate for the betterment of our students. GCTM has been working behind the scenes for a long time to help make this transition as smooth as possible, and we will continue to be a source of strength, professional development, and encouragement for years to come.

As you reflect on your learning, and the growth of your students, please consider writing down your experiences and sharing them with your peers through *Reflections*. These times are fertile ground for personal growth, reflection, and research as we discover how to help our students learn to be their best mathematical selves within this new context. Join us as a contributor or reviewer to illuminate the struggles and highlights in your community. You help us grow too.

Keep Growing,

Dr. Becky Gammill

Editorial Staff

Dr. Becky Gammill
Publication Editor
gammillgctm@gmail.com

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Email the editor at gammillgctm@gmail.com

SUPPORTING STUDENT TRANSFER SKILLS IN MIDDLE SCHOOL MATH

CHEYENNE HUDSON

Abstract

The purpose of this essay is to review the research on the success of different thinking and learning practices for students. More specifically, this essay takes a look at middle school math students. It will also define what transfer is and look at how transfer skills can promote success in a math class. There is a focus on the growth of a student in mathematical ability and the correlation to their success in other areas. Additionally, there is a strong connection between training students to effectively store knowledge in their working memory, and how they perform when tested on this material. These students have been seen to outperform those who are not trained to think successfully. Additionally, research shows that a student's success in math can lead to success in other areas as they grow in their problem-solving abilities. The research supports the claim that transfer can lead to greater student success in school. This essay concludes that teachers can and should try to implement positive transfer more frequently within their instruction and help train their students to use this strategy on their own. This will not only

lead to a higher success in the class where the transfer skills are encouraged but also has been proven to show growth in all areas of that student's studies.

Introduction

As stated, the topic of this essay is how to support students' transfer skills in a middle school math class. I am a middle school math teacher, and I am going into my third year of teaching. Over the past two years, as well as in my student teaching, I have seen students struggle with grasping mathematical skills as new concepts are taught. I have specifically seen my students struggle with concepts that appear like one another. When this happens, they try to perform "tricks" from ideas they've learned with the new concepts. However, these tricks typically do not work as the concepts lack any correlation. A common example of this in my grade and content area is fraction operations. Students find out how to add and subtract fractions and wrongfully assume that multiplying and dividing will follow the same suit. If I could get my students to transfer their knowledge of these operations with whole numbers to the newer concept of fractions, then their strategies would be successful. The hang-up here is that my students would not think that way as fractions and whole numbers do not look similar, so they could not see the similarities. As I have seen my students' difficulties in applying like concepts across different topics within class, I desire to teach them the correct processing skills and understanding of a topic so that they have solid information to transfer to the next topic.

My goal in researching transfer skills was to find a way to help instill these in my students, so they can be more successful in problem solving. This will not only set them up for success in my class, but for the future math classes they have, and potentially other subjects classes as well. Transfer skills are a tool students can use in any area of study. By correctly using transfer abilities in their thinking, students are more efficiently figuring out different scenarios and making connections in concepts. This essay is meant to confirm those theories, as well as give suggestions on how to approach teaching students to work and think about this within one's classroom.

Review of Research

Theme 1: Correlation of Learning and Performance

Transfer is defined as "a phenomenon in which something a person has learned at one time affects how the person learns or performs in a later situation" (Ormrod & Jones, 2023). When looking at students and the material they are learning and studying, transfer refers "not only to how well they can understand and remember it but also to how effectively they can *use and apply* it later on" (Ormrod & Jones, 2023). There is both positive and negative transfer. The difference between the two is if the prior knowledge being pulled helps or harms the student. Sometimes being exposed to material before they are ready can project negative transfer on a student as they are not ready to explore that concept yet. This happens as students either incorrectly apply information they are sorting as prior knowledge, or when students try to make correlations between topics where there is not a relationship. When using transfer skills in teaching and learning, "learners should acquire *conceptual understanding* of a topic, such that many concepts and procedures are interrelated in a cohesive, logical whole" because we know "when knowledge and skills are appropriately interconnected in long-term memory, learners are more likely to retrieve them in relevant situations" (Ormrod & Jones, 2023).

There is an observable correlation between how a student learns content and their performance when tested on that material. Thus, leading to the inference that there are thinking strategies that are specifically aided in student success. This is shown in the research of Mayer and Massa (2003) where the study's goal was to see the correlation between students' achievement and abilities with how they preferred to learn and study. Multiple tests and surveys were given to find any effects. The study's results "clarify the multifaceted nature of the visualizer-verbalizer dimension as consisting of ability, style, and preference subfactors and by suggesting straightforward methods of measuring each subfactor" (Mayer & Massa, 2003). There was a strong correlation between spatial ability measures and how the student was studying. Those who were transferring previous knowledge to their new material correctly and efficiently in their studying were able to recall it easily on assessments, as they had a solidified knowledge of the topic. The study found that verbal-visual learning preference has a distinction from a verbal-visual cognitive style and that the outcomes were surely affected by the procedures in place. Some give credit to this to learning styles, however, suggesting that the students who found success were studying within their learning style, and therefore were more apt to perform well. To look further into the research of learning styles, the article *Prevalence of Learning Styles in Educational Psychology and Introduction to Education Textbooks: A Content Analysis* states that in their review of textbooks, half "defined learning style as a preference or approach, whereas the other half defined it as an individual style" and that the "introduction to education texts tended toward a more positive stance on learning style usage whereas introduction to educational psychology texts exhibited a more neutral stance" (Winninger, Redifer, Norman, & Ryle, 2019).

There is an astounding lack of research to aid in the debate between educators and researchers, on whether or not learning styles have merit. Many educational texts suggest that educators use running styles to aid in their students' studies, including suggestions on how to differentiate this way, but no evidence that it is truly effective.

It will be difficult to suggest that the success of a student is simply due to their studying within a specific learning style. If this is the only strategy used to help students study, "the implementation of learning styles in the classroom could be detrimental to students' encoding, motivation, and self-efficacy" (Winninger, Redifer, Norman, & Ryle, 2019). However, if a teacher looks beyond only differentiating to one's learning style and instead also considers student's differences such as "need for cognition, visual-spatial abilities, open-mindedness, etc.", then educators are focusing more on skills to help students transfer learning successfully, and not just that student's preference on how to study. It is important to note that preference is a huge factor in aiding in student engagement, but it is not the only nor the most important deciding piece in how to help a student grasp material.

There have been numerous studies, proving the effects of transfer and showing how focusing on cognitive learning strategies that dive students deep into the material can help them develop a true understanding of the topics they are studying. One such study showing such is the *Effectiveness of Executive Function Training in Italian Preschool Educational Services and Far Transfer Effects on Pre-academic Skills*. The study sought to "examine the effectiveness and far transfer effects of a training that was found to be effective in promoting Executive Function (EF) in a sample of 5-year-old children" (Traverso, Viterbori, & Usai, 2019).

Teaching students to use executive function means that we want these students to be able to self-regulate and implement cognitive processes that help them think effectively without being instructed to do so eventually. An experimental group was trained on how to transfer, prompting their executive functioning skills, and following the intervention "the experimental group outperformed the control group in an interference suppression composite score" (Traverso, Viterbori, & Usai, 2019). Additionally, "the results suggest the possibility that this intervention, which may be easily implemented in the context of educational services, can promote executive function" (Traverso, Viterbori, & Usai, 2019). As we know, "a key goal of education is enabling learners to transfer their knowledge beyond the initial learning context" (Siler & Willows, 2014). The study by Siler and Willows looks at middle school students and focuses on observing the effects of their transfer skills in mathematics. This study sought to see how introducing new training materials correctly can influence learning and transfer abilities. Its findings suggest that to produce learning and transfer results, there must be a solid interaction between the learner and the task interactions.

Theme 2: Skills to Promote Growth in Mathematic Ability

Math is a subject where the positive transfer of skills from one concept to the next is crucial. There cannot be an understanding of new material without a solid foundation to be built upon. Because of the building block structure of math concepts, transfer is especially important to apply in these classrooms. Evidence of teaching transfer strategies shows that students can grow students' spatial abilities to improve their work in math fields. Specifically, researchers looked at children seeking improvement in their arithmetic skills and gave them mental rotation training (Cheung, Sung, & Lourenco, S., 2020). Control groups were not specifically prompted to use transfer skills in their studying. Throughout a one-week intervention, the study sought to find out if the effect of spatial training transfers to the math domain specifically. The study found mental rotation trainings had significantly proven in their test results, especially in tests that had missing term situations. Additionally, students also showed improvement in areas that the transfer trainings were not even designed for. The research of Gilligan, Thomas, and Farran (2020) looks further into the cause-and-effect relationship between spatial skill development and the improvement of math skills. The student found that spatial training led to "near transfer (to the specific spatial skills trained), intermediate transfer (to untrained spatial skills), and far transfer (to mathematics domains)" (Gilligan, Thomas, and Farran, 2020). These transfer trainings have been proven over and over again to be essential to students' success in math. In yet another study, "following just five days of training, children performed recently trained math problems more efficiently, with greater use of memory-retrieval-based strategies" (Chang, Rosenberg-Lee, Qin, & Menon, 2019).

In studies that only had spatial training and study skills being taught within a week or less, significant growth could already be observed. This can lead to an even bigger impact in students who have deficits in their current math ability. Looking at students with learning disabilities specifically, following the *Pilot Study of an Attention and Executive Function Cognitive Intervention in Children with Autism Spectrum Disorders*, the results showed that the trainings helped students with their working memory and attention. Another important note is that the students transferred these skills and applied them to have gains in math fluency across the board with different concepts. The results of the student clearly "indicate that making math literacy part of a daily routine elicits mathematical development and trains skills that advance students" (Macoun, Schneider, Bedir, Sheehan, & Sung, 2021).

As we know, problem-solving skills that are taught in math can help students become critical thinkers in a multitude of areas. Without the training of transfer skills and productive studying with a mathematics course, students may struggle with these concepts that are not as concrete and miss out on the growth that they could have in other subjects as well. It has been proven in a multitude of studies and classrooms that math literacy is "linked with achievement in math-related domains (ICT literacy, scientific literacy) as well as domains not directly related to mathematics (listening and reading comprehension)" (Holenstein, Bruckmaier, & Grob, 2021). Furthermore, results such as these are what indicate "the existence of a transfer effect of applying this understanding for better life competence in domains other than mathematics" (Holenstein, Bruckmaier, & Grob, 2021).

Conclusion

Research has proven that "learners are much more likely to apply new knowledge and skills when they engage in meaningful rather than rote learning" (Ormrod & Jones, 2023). It is known now that to produce true learning and transfer results, there must be a solid interaction between the learner and the task interactions. It is no mystery that one "key goal of education is enabling learners to transfer their knowledge beyond the initial learning context," and now we see that a major factor "shown to play a role in learning and transfer is the degree of concreteness of instructional materials" (Siler & Willows, 2014). To reap the benefits of transfer within our classroom, regardless of the content we teach, concrete learning must be initiated. Students need to be shown effective study strategies that encourage thinking and connecting concepts within a bigger-picture scenario, and they need to steer away from the outdated practice of memorization. Students must also be taught foundational needs to have something to pull from in their core memory and transfer to new and more challenging scenarios. By starting with simple math literacy skills, cognitive thinking skills can be developed and transfer can begin. There is a plethora of knowledge waiting to be unlocked by your students.

References

Chang, H., Rosenberg-Lee, M., Qin, S., & Menon, V. (2019). Faster learners transfer their knowledge better: Behavioral, mnemonic, and neural mechanisms of individual differences in children's learning. Developmental Cognitive Neuroscience, 40.

Cheung, C.-N., Sung, J. Y., & Lourenco, S. F. (2020). Does training mental rotation transfer to gains in mathematical competence? Assessment of an at-home visuospatial intervention. Psychological Research, 84(7), 2000–2017.

Gilligan, K. A., Thomas, M. S. C., & Farran, E. K. (2020). First demonstration of effective spatial training for near transfer to spatial performance and far transfer to a range of mathematics skills at 8 years. Developmental Science, 23(4).

Holenstein, M., Bruckmaier, G., & Grob, A. (2021). Transfer effects of mathematical literacy: An integrative longitudinal study.

Macoun, S. J., Schneider, I., Bedir, B., Sheehan, J., & Sung, A. (2021). Pilot study of an attention and executive function cognitive intervention in children with autism spectrum disorders. Journal of Autism and Developmental Disorders, 51(8), 2600–2610.

Mayer, R. E., & Massa, L. J. (2003). Three Facets of Visual and Verbal Learners: Cognitive Ability, Cognitive Style, and Learning Preference. Journal of Educational Psychology, 95(4), 833–846.

Ormrod, J. E., & Jones, B. D. (2023). *Essentials of educational psychology: Big ideas to guide effective teaching*. Pearson.

Siler, S. A., & Willows, K. J. (2014). Individual differences in the effect of relevant concreteness on learning and transfer of a mathematical concept. Learning and Instruction, 33, 170–181.

Traverso, L., Viterbori, P., & Usai, M. C. (2019). Effectiveness of an executive function training in Italian preschool educational services and far transfer effects to pre-academic skills.

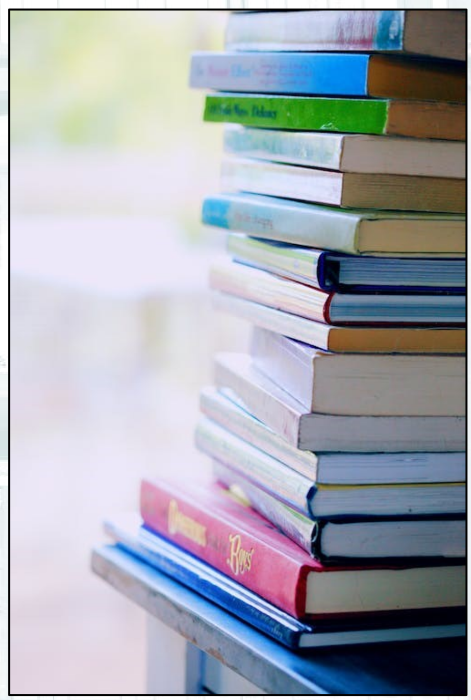
Winninger, S. R., Redifer, J. L., Norman D. A., & Ryle, M. K. (2019). Prevalence of Learning Styles in Educational Psychology and Introduction to Education Textbooks: A Content Analysis. Psychology Learning & Teaching. (Volume 18, Issue 3.)

Cheyenne Hudson is a third-year middle school math teacher in Forsyth County, Georgia. She is currently pursuing her master's in Curriculum and Instruction at the University of North Georgia.

Did You Know?

The Georgia Council of Teachers of Mathematics now offers lists of mathematical literature that may be suitable for your classroom. The resources found on gctm.org are vetted by GCTM members for recommended appropriateness for your classroom. Resources listed are organized by grade level and subject area.

Find out more information at https://www.gctm.org/Mathematics-Literature.



8 MILLION

WES COOPER

Introduction

67,392,000. What might that number represent? The number of miles from the Sun to Venus? The age in years of the Tyrannosaurus rex skeleton known as Sue? Is it the number of Americans per month who receive Social Security benefits? 67,392,000 is the number of seconds a Georgia student spends in school from their first day of kindergarten through high school graduation. Every one of those seconds is an opportunity to be seized or squandered. That same student will spend approximately 8,424,000 seconds in mathematics classrooms learning "to be" (Boaler & Greeno, 2000, p. 188).

As students learn to be, the experiences within a school are pivotal in shaping their mathematics learner identity (MLI) (Anderson, 2007; Barba, 2022) and future career paths (Quintana & Saatcioglu, 2022). Schools communicate to students how well they fit within the mathematics community as individuals and groups (Radovic et al., 2018), with practices likeability tracking playing a significant role in shaping MLI (Trescott, 2020). This tracking can significantly impact students as their "identity beliefs in ninth grade affect their career choices and behaviors" (Quintana & Saatcioglu, 2022, p. 12). Similarly, school climate plays a significant role in MLI development (Jackson et al., 2021) as students engage in a shared experience (Kebreab et al., 2021) and make meaning from that experience. Moreover, student MLI affects "career choices and behaviors" (p. 12), but early interventions can lead to significant changes (Quintana & Saatcioglu, 2022). While schools play a significant role in shaping MLI, I will focus on how teachers shape MLI.

Teachers Shape MLI and Experiences

Teachers play a lifelong role in shaping MLI for students (Varelas et al., 2012). Before discussing this in more depth, it is essential to differentiate between MLI and mathematics teacher identity (MTI) (Graven & Heyd-Metzuyanim, 2019). Darragh (2016) conducted the only comprehensive review on mathematics identity that distinguishes between the two. MTI intentionally and unintentionally shapes MLI (Martin, 2009; Barba, 2022). Teachers are "both architects and stewards" of the communities that shape MLI (p. 8). Teaching practices, teacher expectations, and classroom culture all impact MLI (Anderson, 2007; Parks, 2020; Trescott, 2020). Teacher views and expectations impact the development of MLI (NCTM, 2014) as teachers uniquely define what it means to possess strong, positive MLIs and the purposes of developing those MLIs (Clark et al., 2013). These identities are small and incredibly malleable in the early years and grow more stable over time (Parks, 2020). Kebreab et al. (2021) used Gee's identity framework to explore what they coined pedagogical fluency as "knowing…why and how to choose and enact optimal pedagogical tools to best meet the needs of students. .Pedagogical fluency comes from the knowledge that students' identities are complex, multifaceted, and varied" (p. 949). Just as teachers are shaping MLI, teachers can tap into student identity to facilitate meaningful, contextualized mathematical understanding (NCTM, 2014).

Classroom practices, along with knowledge, engage in an interactive process of shaping MLI (Boaler, 2002). Teachers can offer students opportunities to work through challenging problems that promote reasoning and building mathematical connections, reassuring students of their agency (Barba, 2022). Kebreab et al. (2021) outline an instructional approach that relies on the teacher's knowledge of the content and his or her students. Likewise, teachers can build interest and motivation by providing opportunities for engagement and discourse to help students see the relevance of mathematics to their lives (Miller & Wang, 2019). Anderson (2007) describes this blend of interest and motivation as the alignment face of MLI, where students consider the effort required for mathematics and how it fits within their desired identity. Similarly, teachers can use their knowledge of individual students, their aspirations, and positive relationships to ascertain what might motivate them to enroll in more mathematics opportunities, such as advanced courses or extracurricular mathematics (Anderson, 2011).

In his book Mathematics for Human Flourishing, Su (2020) goes as far as to say that all people are mathematics teachers through our conversations about mathematics. In some settings, individuals might belittle mathematics' utility and relevance, or some may communicate that their family member is *not good at math*. These *teachers* convince young people that mathematics is not a space for them.

Goldin et al. (2016) noted how students developed different MLI based on which of two learning environments they were in (Bishop, 2012; Fernández et al., 2022), like the two pedagogical discourses identified by Heyd-Metzuyanim and Shabtay (2019). The first was the traditional mathematics path of teacher-centered, procedural fluency, and the second was the reformed path of student-centered, conceptual understanding (Boaler, 1997; Senk & Thompson, 2003). Mainly based on student experiences in K12 classrooms, these two paths have distinct outcomes with which they are associated. At this point, I will discuss these two distinct approaches within mathematics learning environments.

Emphasis on Procedural Fluency

Procedural fluency has long been an approach of mathematics classrooms, with timed operation fluency assessments across the country alongside games rewarding speed. Graven and Heyd-Metzuyanim (2019) identified this as acquisition-based teaching, while Friere (2017) identified this as the banking model of education.

While not all classrooms implementing these practices focus exclusively on procedural fluency, some settings predominantly focus on this repetitive practice (Boaler & Greeno, 2000), which was how they often experienced math education as students (Ainsworth & Christinson, 2006). An emphasis on high-stakes testing further encourages teachers to focus on computational skills and exclude problem-solving skills. The emphasis on procedural fluency in K12 is partly due to the importance placed on this by some university mathematicians (Boaler & Greeno, 2000). While students who view mathematics as a set of procedures tend to appreciate the right or wrong nature of mathematics, they also tend to view mathematics in direct conflict with their identity development. Since mathematics does not allow exploration, it is mutually exclusive of the person they want to become– "creative, verbal, and humane" (p. 187). It should come as no surprise that these classrooms tend to follow a didactic nature, which I will discuss next.

Math as Didactic.

Communication follows a didactic nature in settings that emphasize the procedural aspects of mathematics. This approach presents mathematics as a performance delivered to an audience of learners. This communication style frames math as merely a process of receiving knowledge (Boaler, 2002; Boaler & Greeno, 2000), which sets the teacher atop the classroom hierarchy as the controller of knowledge (Anderson, 2011; Boaler & Greeno, 2000), whereby the teacher deposits knowledge into the bank of each student's mind (Freire, 2017). Since learning in these classrooms is not believed to occur without the teacher leading the way, the success of these classrooms is often dependent on compliance (Boaler & Greeno, 2000). Students in didactic, procedurally focused classes follow along as the instructor demonstrates a set of steps to follow (Anderson, 2011). These students often do not understand the why and how behind mathematical concepts (Boaler & Greeno, 2000). Consequently, deficiencies in the didactic approach often lead to students having less mathematical agency (Boaler & Greeno, 2000, p. 195). I will now discuss how the procedural approach paints math as rigid and unimaginative.

Math as Rigid and Unimaginative.

The procedural approach to math education focuses on memorized steps at the expense of understanding (Hughes, 2009), viewing math as something one tells someone else (Su, 2020). The procedural approach often leads to learners who dislike and disengage with math in exchange for courses that provide space for expression (Barba, 2022). Students have indicated they feel less able to be "thoughtful or creative" in mathematics classrooms emphasizing procedural fluency (Boaler & Greeno, 2000, p. 188). However, these students might engage passively (Boaler & Selling, 2017) with mathematics and test well with minimal understanding, leading some to view the procedural approach as a success and others to view the procedural approach as a failure (Schoenfeld, 1988). At this point, I will discuss how the procedural approach leads to a passive view of mathematics.

Math as Passive.

Along with mathematics as didactic, mathematics, when approached procedurally, is often viewed as passive. University students have complained of undergraduate students being dependent on "spoon-fed" (Boaler & Greeno, 2000, p. 196), step-by-step instructions, a logical development from the didactic approach that positions students as disengaged, received knowers even in the most advanced high school mathematics courses. In situations where students positively identified with mathematics in the procedurally focused setting, students attributed this to how mathematics "allowed them to passively receive knowledge" (Boaler & Greeno, 2000, p. 188). Thus, these students often lack agency when given novel problems (Lave, 1988). Likewise, the procedural approach often contributes to a fixed mindset, which I will discuss now.

Fixed Mindset.

Mathematics teaching from a traditional, procedural approach often alienates students (Boaler, 2002). To counter this, Anderson (2007) calls for teachers to discount the nature component of identity, which consists of things determined by nature, such as gender and race, which might consciously or subconsciously lead to inequitable outcomes. Many students have come to view their own mathematical identities in binary, permanent terms: easy or hard, quick or impossible (Darragh, 2013b); Boaler (2019) dismisses this binary view of mathematics with a vast amount of research. Students who held a fixed mathematical mindset "viewed efficacy as a measure of intelligence, emphasized speed over effort, [and] scrutinized the problem, such as devaluing or debunking it, to preserve their perceived rank" (Barba, 2021, p. 28). With the procedural approach, the emphasis is on responding correctly with canned responses rather than collaborating to grow as a community of learners and doers of mathematics (Darragh, 2013b; Gardee & Brodie, 2023). The fixed mindset brings views of superiority, authority/power, and inferiority (Barba, 2021). If students do not experience success early, the self-narratives they tell themselves are often stories of failure as outsiders to mathematics. On the other hand, if they experience success early, they can view others as less worthy and able, destined for a life of mathematical inadequacy. Students from this perspective often view mathematics with a simplified perspective of right or wrong rather than working toward an improved understanding of phenomena and persevering through problem-solving (Darragh, 2013b). Like the fixed mindset, the procedural approach also portrays math as exclusionary, which I will discuss at this point.

Math Education as Exclusionary.

The procedural approach to mathematics instruction positions students within the circle of mathematics or outside with no ability to enter (Solomon, 2007). Without the creative, exploratory approaches to mathematics that foster connectedness and understanding, students are often disenfranchised from mathematics as they develop negative MLIs (Solomon, 2007; Burton, 1998) based on positions of power, superiority, and inferiority (Barba, 2021). In settings where mathematics is taught primarily through a procedural lens, the content becomes one with which students do not need to engage actively, which contrasts with the conceptual approach I will discuss next.

Emphasis on Conceptual Understanding

Graven and Heyd-Metzuyanim (2019) identified an exploratory-based teaching approach. Friere (2017) identified this as the mining model of education. This approach involves open-ended questions where teachers pull knowledge out of students and rely on intentional questioning with elevated rigor to build conceptual understanding (Dougherty et al., 2015). Much of the literature on approaching mathematics from a conceptual standpoint contrasts significantly with the literature from a procedural standpoint. For example, students who view mathematics from a conceptual perspective rather than a procedural one tend to have more appreciation for the creative, exploratory aspects of problem-solving and the beauty of mathematics (Ainsworth & Christinson, 2006; Boaler & Greeno, 2000). Often, the high expectations that come with teaching for conceptual understanding lead to higher achievement (NCTM, 2014) by engaging students in the learning process through interactive assessment and practices (Silver et al., 2009). This exploratory approach to mathematics helps form strong mathematical identities built on constant participation, exploration, and dialog, which together foster agency (Solomon, 2007). The conceptual approach is built on dialog, which I will discuss now.

Math as Dialogical.

In contrast to the procedural view of education as banking, the conceptual path frames education as mining, centering students in the learning process based on the value students bring in their approaches to mathematics (Anderson, 2011; Freire, 2017). With the conceptual approach, educators are pulling the mathematics out of their students, strategically facilitating mathematical community development (Barba, 2022) based on contributing through collaborative problem-solving rather than merely answer-sharing (Darragh, 2013a). Through discursive classroom practices (Kebreab et al., 2021), these dialogs build community and classroom connectedness (Burton, 1998) as students develop a belief that engaging in mathematical discussion benefits the entire class. Number Talks, introduced in the 1990s, is one example of this conceptual, dialogical approach (Humphreys & Parker, 2015). Through this exploration and the accompanying discussions, MLI is built by further advancing proofs as arguments are made and shared with the greater mathematics community (Boaler & Greeno, 2000). In these non-didactic settings, students are encouraged to develop questions and assess the validity of ideas, similar to the peer-review process (Anderson, 2007; Boaler & Greeno, 2000). However, these spaces are socially risky as they require the learner to be vulnerable (Horn, 2017). This questioning and justifying mathematical reasoning allow students to grow stronger MLIs (Boaler & Greeno, 2000). These dialogs are not exclusively interpersonal. Consequently, discovery can be in the form of social interaction or individual struggle since this dialog can be interpersonal or intrapersonal (Boaler & Greeno, 2000). Through discourse, these students construct and reconstruct their identities (Heyd-Metzuyanim & Shabtay, 2019). An emphasis on conceptual understanding allows a richer dialog with the math concepts themselves as the curriculum can be organic, responding to students much like Nasir's (2002) approach to infusing the learning of mathematics through dominoes and basketball into the classroom to "solve authentic problems, messy ones without clear right or wrong answers, perhaps in the service of a nonmathematical goal" (p. 242). In solving problems leading to a goal apart from solely solving the problem, students develop a mathematical identity built on agency. I have discussed how the conceptual approach to math is built on dialog, and now I will discuss how this approach views math as creative.

Math as Creative.

Mathematicians often describe mathematics as playful, beautiful, and inherently creative (Zager, 2017), built on wonder and exploration (Boaler & Dweck, 2016). When teachers emphasize a conceptual approach through consistent, open-ended questioning with nonroutine tasks, students can appreciate the creative aspects of mathematics (Anderson, 2011; Aungst, 2016; Boaler, 2013; Brahier, 2016; Leinwand, 2009). In this approach, students can explore and develop their strategies (Anderson, 2007) through discovery (Barba, 2022) as the teacher fosters intuition and creativity (Boaler & Dweck, 2016; Burton, 1999). This teaching approach provides students with moments for genuine discovery (Su, 2020) through independent investigations (Skilling, 2014). Teaching strategies that show mathematics as creative (Boaler, 2013) and fun contribute to improved engagement and formation of positive, confident, and persistent MLIs (Gweshe & Brodie, 2019). I have discussed how the conceptual approach to mathematics views math as creative. Now, I will discuss how this approach contributes to student agency.

Agency.

Students who view mathematics from a conceptual lens tend to view mathematics as an empowering tool by which they embrace agency (Trescott, 2020) through "validating mathematics methods, generating questions, and developing ideas" (Barba, 2022, p. 11). Although some students authored identities that overcame deficiencies in the procedurally focused environment (Boaler & Greeno, 2000), teachers can facilitate this by building classroom structures and redirecting students to the learning goals and expectations (Barba, 2022).

When meaningful, real-world experiences (Anderson, 2007; Eraso, 2013; Ryg, 2014) undergird mathematics instruction, students can see themselves as active participants in the mathematics community (Miller & Wang, 2019). Students develop more positive mathematics identities when they engage thoughtfully with mathematics and "develop connected, relational understanding" (Boaler & Greeno, 2000, p. 188). Therefore, a conceptual view of mathematics equips students with an ability to engage in interpersonal and intrapersonal dialog and embrace agency to respond to questions and problems. I have discussed how the conceptual approach to mathematics fosters agency in students. Now I will discuss how a growth mindset is necessary in the conceptual approach.

Growth Mindset.

The conceptual approach to mathematics instruction helps students embrace the inherent mistakes of being a learner and doer of mathematics (Zager, 2017). Mathematics students with a growth mindset: "viewed efficacy as distinct from intellectual capacity, sought positive interpretations of their failure, valued effort over speed, [and] were disinterested in their perceived rank" (Barba, 2021, p. 28). A focus on grades and a spirit of competition is in opposition to a growth mindset built on collaboration and growing as a learner and doer of mathematics (Gardee & Brodie, 2023). This growth mindset contributes to positive MLIs as students learn to value various approaches to mathematics (Barba, 2021). Teachers can support students' MLI by providing opportunities for students to consider their growth as learners and doers of mathematics (Anderson, 2007).

Building reflective learners who believe that everyone can learn math at high levels and that mistakes are a part of the learning process (Boaler & Dweck, 2016) can foster a growth mindset. Students with a growth mindset often view mathematics as complex and multifaceted.

They often see problems as continuums for which they can work toward a better understanding, as espoused in state frameworks for mathematical modeling (GaDOE, 2021). I have discussed how a growth mindset is imperative in the conceptual approach to mathematics as students seek improved understanding instead of speed or hierarchical positions. I will now discuss how the conceptual approach to mathematics is inviting.

Math Education as Inviting.

The conceptual approach to mathematics instruction invites all to think, discuss, create, and solve problems (Solomon, 2007). This approach is evident in the re-emergence of rich mathematics tasks, such as the growth of 3-Act Tasks through math educators like Kristen Acosta, Graham Fletcher, Kendra Lomax, Dan Meyer, and Kyle Pearce. 3-Act Tasks provide a low-entry point with a high ceiling for mathematical understanding. These nonroutine problems welcome many different approaches (Brahier, 2016) in which the educator utilizes intentional questions to build understanding (Barba, 2022). In the conversation, everyone can contribute to the mathematics community's understanding of the scenario. The classroom becomes an inviting space, with everyone on an even playing field, welcoming everyone to contribute. As outlined in Georgia's K12 mathematics standards, mathematical modeling provides another avenue for welcoming students into mathematical thinking, as do the statistical reasoning frameworks (GaDOE, 2021).

Conclusion

Despite not knowing you, I am confident in something. I am confident of your heart for students: the desire to help them reach their potential, to watch them learn and grow as students and people who are problem-solvers. I am convinced that to help students reach their mathematical potential as problem-solvers, we should be teaching for conceptual understanding, and building strong, positive MLIs. However, to help accomplish these things, students must see mathematics from a conceptual orientation across the entirety of their mathematics education experiences. As math leaders, we must help others see the value in a conceptually focused instructional approach to mathematics as we help students see themselves as learners and doers of mathematics. I am grateful to have colleagues like yourself to partner with in this process as we work to make those 8 million seconds as impactful as possible.

References

Ainsworth, L., Christinson, J. (2006). Five Easy Steps to a Balanced Math Program for Secondary Grades. United States: Advanced Learning Press.

Anderson, R. (2007). Being a mathematics learner: Four faces of identity. *The Mathematics Educator, 17*(1), 7—14. https://files.eric.ed.gov/fulltext/EJ841557.pdf

Anderson, R. (2011). What motivates students to take advanced mathematics: A rural perspective. In D. J. Brahier (Ed.), *Motivation & disposition: Pathways to learning mathematics*. Reston, VA: National Council of Teachers of Mathematics.

Aungst, G. (2016). *5 principles of the modern mathematics classroom*. Corwin, https://dx.doi.org/10.4135/9781483394817

Barba, K. (2021). The mathematical mindsets and mathematical identities revealed in social media discourse. *Journal of Mathematics Education at Teachers College*, *11*(2), 23–34. https://doi.org/10.7916/jmetc.v11i2.7833

Barba, K. M. (2022). Mathematical identity and the role of the educator. *Journal of Mathematics Education at Teachers College*, *13*(1), 7–13. https://doi.org/10.52214/jmetc.v13i1.9187

Bishop, J. P. (2012). "She's always been the smart one. I've always been the dumb one": Identities in the mathematics classroom. *Journal for Research in Mathematics Education*, *43*(1), 34–74.

Boaler, J. (1997). *Experiencing school mathematics:Teaching styles, sex and setting.* Open University Press.

Boaler, J. (2002). The development of disciplinary relationships: Knowledge, practice and identity in mathematics classrooms. *For the Learning of Mathematics*, *22* (1), 42 – 47.

Boaler, J. (2013). Ability and mathematics: The mindset revolution that is reshaping education. *Forum, 55*(1), 143–152.

Boaler J. (2019). *Limitless mind: Learn, lead, and live without barriers*. Harper Collins Publishers.

Boaler, J., & Dweck, C. S. (2016). *Mathematical mindsets: Unleashing students' potential through creative math, inspiring messages and innovative teaching.* Jossey-Bass.

Boaler, J., & Greeno, J. G. (2000). Identity, agency, and knowing in mathematics worlds. In J. Boaler (Ed.), *Multiple perspectives on mathematics teaching and learning* (pp. 171–200). Ablex.

Boaler, J., & Selling, S. K. (2017). Psychological imprisonment or intellectual freedom? A longitudinal study of contrasting school mathematics approaches and their impact on adults' lives. *Journal for Research in MathematicsEducation, 48*(1), 78–105.

Brahier, D. J. (2016). Teaching secondary and middle school mathematics. Routledge.

Burton, L. (1998). The practices of mathematicians: What do they tell us about coming to know mathematics? *Educational Studies in Mathematics, 37*(2), 121–143.

Burton, L. (1999). Exploring and reporting upon the content and diversity of mathematicians' views and practices. *For the Learning of Mathematics*, *19*(2), 36–38.

Clark, L. M., Badertscher, E. M., & Napp, C. (2013). African american mathematics teachers as agents in their african american students' Mathematics Identity Formation. *Teachers College Record, 115*(2), 1–36.

Darragh, L. (2013a). Constructing confidence and identities of belonging in mathematics at the transition to secondary school. *Research in Mathematics Education, 15*(3), 215–229.

Darragh, L. (2013b). In V. Steinle, L. Ball & C. Bardini (Eds.), Mathematics education: Yesterday, today and tomorrow (Proceedings of the 36th annual conference of the Mathematics Education Research Group of Australasia). Mathematics Education Research Group of Australasia Inc. 2013

Darragh, L. (2016). Identity research in mathematics education. *Educational Studies in Mathematics, 93*(1), 19–33. https ://doi. org/10.1007/s1064 9-016-9696-5.

Dougherty, B., Bryant, D. P., Bryant, B. R., Darrough, R. L., & Pfannenstiel, K. H. (2015). Developing concepts and generalizations to build algebraic thinking: The reversibility, flexibility, and generalization approach. *Grantee Submission*, *50*(5), 273–281.

Eraso, M. (2013). Adolescents learn addition and subtraction of integers using algebra project's curriculum process. In J. Leonard & D. B. Martin (Eds.), *The brilliance of Black children in mathematics: Beyond the numbers and toward new discourse* (pp. 151–170). Information Age Publishing.

Fan, X., & Chen, M. J. (1999). Academic achievement of rural school students: A multi-year comparison with their peers in suburban and urban schools. *Journal of Research in Rural Education, 15*, 31–46.

Fernández, L. M., Nguyen, U., & Callahan, R. (2022) Learners' mathematics identity and achievement: Where does the teacher come in?, *International Journal of Mathematical Education in Science and Technology*, DOI: 10.1080/0020739X.2022.2117657

Freire, P. (2017). *Pedagogy of the oppressed*. Penguin Classics.

GaDOE (2021). Statistical Reasoning Framework. https://www.gadoe.org/Curriculum-Instruction-and-Assessment/Curriculum-and-Instruction/Documents/Mathematics/Georgia-K12-Mathematics-Standards/Georgia-HS-Statistical-Reasoning-Mathematics-Standards.pdf

Gardee, A., Brodie, K. (2023). A framework for analysing the relationships between peer interactions and learners' mathematical identities: accounting for personal and social identities. *Educational Studies in Math*. https://doi.org/10.1007/s10649-022-10201-0

Goldin, G. A., Hannula, M. S., Heyd-Metzuyanim, E., Jansen, A., Kaasila, R., Lutovac, S., Di Martino, P., Morselli, F., Middleton, J. A., Pantziara, M., Zhang, Q. (2016). *Attitudes, beliefs, motivation and identity in mathematics education : An overview of the field and future directions* . Cham, Switzerland: Springer Open.

Graven, M., & Heyd-Metzuyanim, E. (2019). Mathematics identity research: the state of the art and future directions. *ZDM, 51*, 361—377.

Gweshe, L. C., & Brodie, K. (2019). High school learners' mathematical identities. *African Journal of Research in Mathematics, Science and Technology Education, 23*(2), 254—262.

Heyd-Metzuyanim, E., & Shabtay, G. (2019). Narratives of 'good' instruction: Teachers' identities as drawing on exploration vs. acquisition pedagogical discourses. *ZDM - Mathematics Education, 51*(3), 541– 554. https://doi.org/10.1007/s11858-018-01019-3

Horn, I.S. (2017). *Motivated: designing math classrooms where students want to join in*. Heinemann

Hughes, G.B. (2009). Students' perceptions of teaching styles in mathematics learning

environments. *Mathematics Teaching-Research Journal Online, 3*(2), 1—12.
Humphreys, C., Parker, R. E. (2015). Making Number Talks Matter: Developing Mathematical Practices and Deepening Understanding, Grades 4-10. United States: Stenhouse Publishers.
Jackson, L., Ford, J., Randolph, C., Schleiden, C., Harris-McKoy, D., & McWey, L. (2021). School climate as a link between high school black males' math identity and outcomes. *Education and Urban Society, 53*(4), 469–487. https://doi.org/10.1177/0013124520931453
Kebreab, L., Bush, S. B., & Jackson, C. (2021). Leveraging student identities to develop pedagogical fluency, *Mathematics Teacher: Learning and Teaching PK-12*, *114*(12), 948—955.
Lave, J. (1988). *Cognition in practice*. Cambridge: Cambridge University Press.
Leinwand, S. (2009). *Accessible mathematics: 10 instructional shifts that raise student achievement* . Heinemann.
Martin, D. B. (2009). Researching race in mathematics education. *Teachers College Record, 111*(2), 295–338.
Miller, R. S., & Wang, M.T. (2019). Cultivating adolescents' academic identity: Ascertaining the mediating effects of motivational beliefs between classroom practices and mathematics identity. *Journal of Youth and Adolescence, 48*(10), 2038–2050.
Nasir, N. I. S. (2002). Identity, goals, and learning: Mathematics in cultural practice. *Mathematical thinking and learning*, *4*(2-3), 213-247. https://doi.org/10.1207/S15327833MTL04023_6
National Council of Teachers of Mathematics. (2014). *Principles to actions: Ensuring mathematical success for all*. NCTM.
Parks A. N. (2020). Centering children in mathematics education classroom research. *American Educational Research Journal, 57*(4), 1443–1484. https://doi.org/10.3102/0002831219873853
Quintana, R., & Saatcioglu, A. (2022). The long-lasting effects of schooling: Estimating the effects of science and math identity in high school on college and career outcomes in stem. *Socius*, *8*, 1—14. https://doi.org/10.1177/23780231221115405
Radovic, D., Black, L., Williams, J., & Salas, C. (2018). Towards conceptual coherence in the research on mathematics learner identity: A systematic review of the literature. *Educational Studies in Mathematics*, *99*(1), 21—42. https://doi.org/10.1007/s10649-018-9819-2
Ryg, M. A. (2014). Radical equations as american philosophy: Dewey's experience, quine in a straight jacket, and algebra from moses. *Contemporary Pragmatism*. Brill Academic Publishers. https://doi.org/10.1163/18758185-90000288
Schoenfeld, A.H. (1988). When Good Teaching Leads to Bad Results: The Disasters of "Well-Taught" Mathematics Courses. *Educational Psychologist, 23*(2), 145—166.
Senk, S.L., & Thompson, D.R. (Eds.). (2003). Standards-based school mathematics curricula: What are they? What do students learn? (1st ed.). Routledge. https://doi.org/10.4324/9781003064275
Silver, E. A., Mesa, V. M., Morris, K. A., Star, J. R., & Benken, B. M. (2009). Teaching mathematics for understanding: An analysis of lessons submitted by teachers seeking nbpts certification. *American Educational Research Journal, 46*(2), 501–531. https://doi.org/10.3102/0002831208326559
Skilling, K. (2014). Teacher practices: How they promote or hinder student engagement in mathematics. In J. Anderson, M. Cavanagh, & A. Prescott (Eds.), Curriculum in focus: research guided practice. Proceedings of the 37th annual conference of the Mathematics Education Research Group of Australasia (pp. 589–596). MERGA

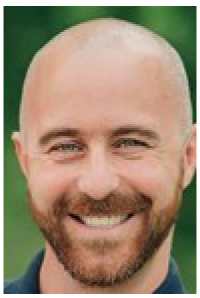
Solomon, Y. (2007). Not belonging? What makes a functional learner identity in undergraduate mathematics? *Studies in Higher Education*, *32*(1), 79–96. doi:10.1080/03075070601099473

Su, F. E. (2020). *Mathematics for human flourishing*. Yale University Press.

Trescott, Ann, "The Construction of Student Mathematical Identity and its Relationship to Academic Achievement" (2020). Dissertations. 180. https://digital.sandiego.edu/dissertations/180

Varelas, M., Martin, D., & Kane, J. M. (2011, June). Content learning and identity construction (CLIC): An interpretive framework to strengthen African American students' mathematics and science learning in urban elementary schools. Symposium presented at the annual meeting of the Jean Piaget Society, Berkeley, CA.

Zager, T. J. (2017). Becoming the math teacher you wish you'd had. Stenhouse Publishers.



Wesley Cooper serves as the K-12 Math Coordinator for Morgan County Charter School System and the Academic Coach for Morgan County Middle School. He is completing his Ed.D. in Secondary and Middle Grades Mathematics at Kennesaw State University where he is conducting research on mathematics learner identity and mathematical experiences. He wants all students to have strong, positive mathematical learner identities, and he enjoys helping students build these identities through accessible, engaging tasks and experiences.

FUN FACTS ABOUT 8,000,000

According to the Atlanta Business Chronical, the population of Atlanta will reach 8,000,000 by 2040.
Source:
https://atlurbanist.tumblr.com/post/128856184249/what-does-a-place-with-8-million-people-look-like

What's the difference between a million, a billion, and other large numbers?

Find out at the link below.

Q: What galaxy is 8 million light years away?
A: NGC 2403. Find out more about this Galaxy here: https://cs.astronomy.com/asy/m/galaxies/493980.aspx

Irrational numbers don't have to be irrational

Joseph Melvin Kelly, Ed.D.

When asked what is the square root of 16, some might quickly say they have no idea, while others might incorrectly say eight, and others might correctly respond four. For those who answer correctly, the next question, of why four is the square root of 16, leads to a deeper and more fruitful conversation on mathematics education.

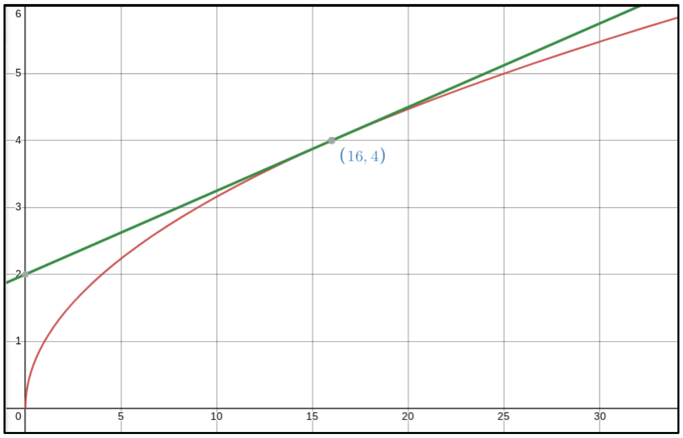
Conceptual understanding of irrational numbers, especially square roots, is foundational to the study of more advanced mathematical topics. Just as with an understanding of fractions, students sometimes feel intimidated and have a limited understanding of the set of irrational numbers. While students can quickly learn how to calculate the square root of a number using any standard scientific calculator, a deeper understanding leads to the ability to reason numerically and better approximate values of irrational numbers. Under Georgia's K-12 mathematics standards, students in grade eight math must distinguish between rational and irrational numbers using decimal expansion, convert a repeating decimal expansion into a rational number, approximate irrational numbers to compare the size of irrational numbers, locate irrational numbers approximately on a number line, and estimate the value of rational numbers. The following section further discusses various methods used in approximating the square roots of non-perfect squares.

Methods for Approximation

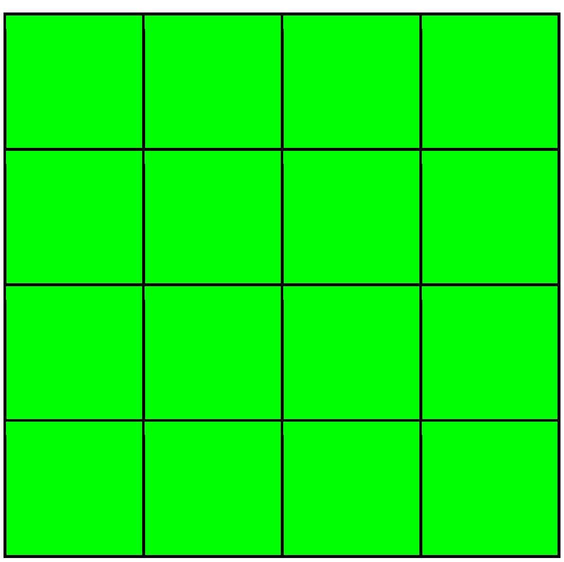
Throughout history, civilizations have strived to develop better approximation methods for calculating the square root of a non-perfect square. The Greeks are typically credited with the first formal proof of the square root of two being an irrational number (Edgett, 1935). The Babylonians, Chinese, and Egyptians each developed their approximation methods. One of the simplest means of approximating irrational numbers is simply looking at perfect squares. For example, if we wanted to approximate the square root of 17, we could reason that since the square root of 16 is four, and the square root of 25 is five, then the square root of 17 is approximately between four and five. While this provides a quick means of approximation, it lacks efficiency and precision.

Newton's method for a tangent line approximation provides a means of utilizing Calculus to help approximate the values of square roots of non-perfect squares. This method works because it approximates a general function using a linear function and the point-slope form $y - y_1 = m(x - x_1)$ where m represents the derivative of the function at the point (x_1, y_1). For example, suppose we wanted to approximate the square root of 17. First, we would let $f(x) = \sqrt{x}$. Next, we would identify the point (16, 4) as being on the graph of $f(x)$. To approximate the value of $f(17)$, we would apply the linear approximation $y - 4 = f'(16)(17 - 16)$. This leads us to $y = \frac{1}{8}(1) + 4$. Therefore, the square root of 17 is approximately $4\,\frac{1}{8}$ or 4.125 using this method. While this approximation is an overestimate of the true value $4.1231\ldots$, the approximation is correct to two decimal places. While not appropriate for a grade eight class, this approximation method does allow students taking Calculus to see an application of Newton's tangent line approximation method and practice of calculating the derivative of a function at a specified point.

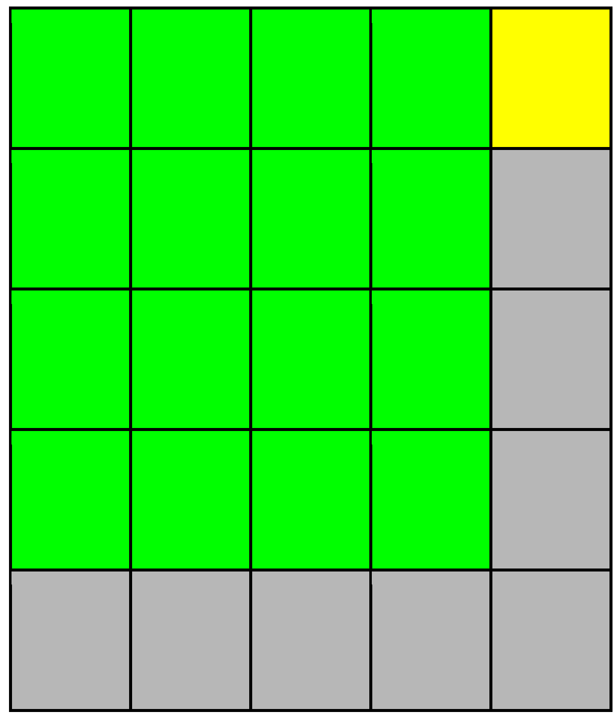
It is important to note that Newton's method works best for approximations close to a known point on the function. Therefore, as shown in the following image, the precision decreases as the distance from a perfect square increases.



A different strategy that utilizes a geometric approach could potentially allow students to develop a better conceptual understanding of the square root of a number with the help of visualization. Unit blocks provide such a means to visualize square roots. For example, students can visualize an area of 16 with 16 unit blocks. Students can arrange these 16 blocks to form a square with side lengths of four. Doing so allows students to visualize the square root of 16 being equal to four because it represents the side length of a square with an area of 16.



Interestingly, students can utilize this same method to visualize the square root of non-perfect squares and approximate the values of irrational numbers. Suppose we wanted to find the square root of 17. We could arrange 17-unit blocks to almost form a perfect square. We would have a smaller perfect square with a side length of four and one square leftover with eight squares missing to complete the larger perfect square.



Therefore, the approximation for the square root of 17 would be $4\,\frac{1}{9}$ or $4.111\ldots$. Notice this method produces an underestimate of the true value of $4.1231\ldots$ and is correct to one decimal place while not relying on differential Calculus.

Comparing the two methods of linear approximations and unit blocks with other irrational numbers provides some interesting results. Suppose we wanted to approximate the square root of 24, using Newton's method, we would get five.

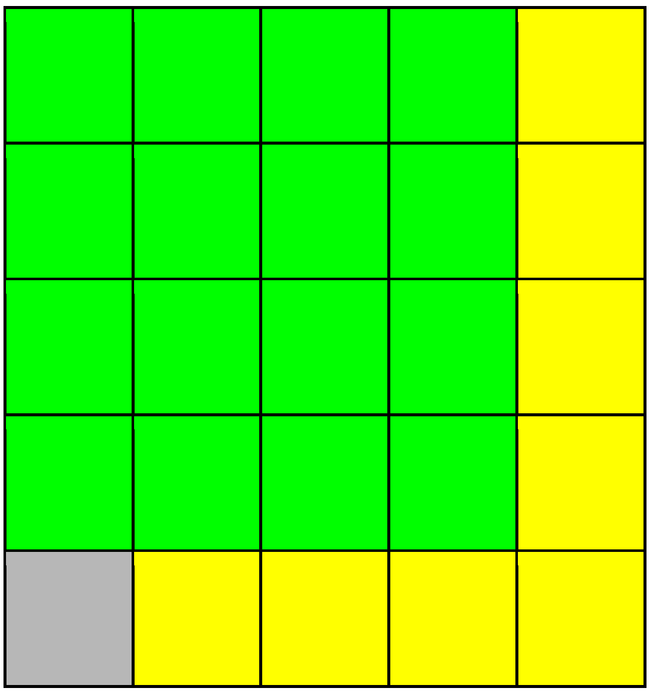
$$y - 4 = f'(16)(24 - 16)$$

$$y = \frac{1}{8}(8) + 4$$

$$y = 5$$

We notice this approximation is not precise since the square root of 25 is five so the square root of 24 must be less than five.

Using the unit blocks method, we get approximately $4\ \frac{8}{9}$ or $4.888\ldots$ compared to the true value of $4.898\ldots$.



The unit blocks method provides a means to improve precision, while also allowing students to visualize the square root of a nonperfect square. The next section discusses the methodology of this qualitative study by investigating the influence of using the unit blocks method to help students visualize and approximate irrational numbers.

The Task

To better understand how students conceptualize the square root of a number, a sample consisting of two algebra teachers and two algebra students was taken. Each participant took part in a learning task involving approximations of the square roots of non-perfect squares utilizing unit blocks. As each participant worked through the task, observation notes were taken on how students interacted with the activity.

Learning Task

Today, we are going to be looking at the square roots of numbers without using technology.

1. What is the square root of 16? Why?
2. What is the square root of 25? Why?
3. What is the square root of 21?
4. How is question 3 different from questions 1 and 2?
5. We are now going to use unit blocks to help us visualize these numbers. Take 16 unit blocks and arrange them to make a square. What do you notice about the lengths of the sides? How does this relate to your answer to question 1?
6. Take 25 unit blocks and arrange them to make a square. What do you notice about the lengths of the sides? How does this relate to your answer to question 2?
7. Take 21 unit blocks and arrange them to make a square. What is the issue?
8. Notice we do have a smaller square with 5 extra block tiles. How many more unit blocks would you need to form the big square?
9. Let's represent the “extra blocks” as a fraction of the total number of blocks needed to make the next square. What would this fraction be?
10. Using the length of the smaller square and the fraction you created, what would be your approximation for $\sqrt{21}$?
11. Using the block tiles, approximate the following square roots:

    a. $\sqrt{10}$ b. $\sqrt{32}$.

Following the learning task, each participant also responded to a survey.

Survey

1. How do you describe the square root of a number?

2. What are the advantages of using unit blocks to approximate square roots of non-perfect squares?
3. What are the disadvantages of using unit blocks to approximate square roots of non-perfect squares?

Results

To better understand how students conceptualize square roots, observation notes, and interview data were collected. Each participant worked through the learning task. All four participants were engaged in the activity and verbally expressed interest and amazement while working through the mathematics. After using the unit blocks to approximate square roots, one of the teachers responded, "Get out of here! Will this always be precise to one decimal place?" Wanting to double-check his arithmetic, one of the students utilized the calculator on his cellphone to verify his approximations. His amazement and engagement increased as he continued using unit blocks to make more approximations. In addition, the other student and teacher deviated from using the unit blocks for the approximations and instead started drawing squares and blocks on paper. This showed the adaptability of the participants to utilize different approaches while working through the activity.

Following the learning task, each participant's conceptualization of the square root of a number shared a similar geometric theme. In their descriptions, phrases such as "the square root being equal to side length", "the number used to get a square", and "dividing into two equal parts" were used. While the latter comment was not precise, the student further elaborated that "the square root of 16 is four because 16 can be divided into two equal parts of four." Although division would not be an appropriate terminology to use, the student leaned on a geometric perspective when describing the square root of a number.

Additionally, all four participants shared perspectives on the advantages of using unit blocks to approximate square roots. One teacher noted the importance of visualization in that you "can see the perfect square." One of the students shared a similar thought in that he was able to "see the blocks and see the math." The other student noted how this learning task incorporates multiple representations, "It shows you other ways to solve square roots instead of just basic math." The other teacher expressed excitement in "showing my students this method to strengthen the geometric connection when completing the square to find equivalent expressions with quadratics and conic sections." While each participant shared many advantages to the learning task, there were also suggested disadvantages to the activity. One teacher stated, "Larger numbers such as fifty would require lots of squares." A student shared a similar argument in that, "it is more time consuming and will take up more space." Suggestions to address these comments are found in the next section including a discussion of the results.

Discussion

The results from this study provide practical and relevant implications for the algebra classroom. Using unit blocks to approximate square roots provides a learning task that supports visual mathematics and multiple representations. Visual mathematics is an evidenced-based instructional practice used to support the learner's mathematical growth and development. Boaler et al. (2016) indicate that the human brains think about mathematics visually and students who see mathematics visually understand the concepts more deeply. As evident in the interview responses, all participants in the study noted the advantage of using unit blocks to visually see the mathematics and approximations of the square roots or rational and irrational numbers.

Additionally, participants referenced the learning task's use of multiple mathematical representations. Mathematical representations, such as unit blocks, are visible and tangible tools accessible to students. The National Council of Teachers of Mathematics (2014) identifies the use and connection of mathematical representations as an effective mathematics teaching practice. Implementing this practice in the classroom allows students to deepen their understanding of mathematics concepts and procedures. Participants in the study articulated a geometric perspective toward the conceptualization of square roots after completing the learning task. Being able to visually see the connection between an abstract irrational value and concrete approximation for the side length of a square allows students a means to represent mathematics in multiple ways.

Finally, the participants also noted the disadvantages of the learning task including the method for approximation being time-consuming and requiring a lot of blocks for larger values. The implementation of technology could potentially alleviate these issues. As always, teachers should evaluate and utilize curriculum resources based on the overall needs of their students and classes, modifying them as needed. While teachers may not feel comfortable in implementing the task without further professional development and practice, the benefits of utilizing unit blocks to approximate irrational numbers allow for opportunities for evidence-based instructional practices in the classroom including visual mathematics and multiple representations.

References

Boaler J., Chen L., Williams C., & Cordero M. (2016). Seeing as understanding: The importance of visual mathematics for our brain and learning. *Journal of Applied and Computational Mathematics*, 5, 325–330. https://doi.org/10.4172/2168-9679.1000325

Edgett, G. L. (1935). The irrational number. *National Mathematics Magazine, 9*(7), 193–196. https://doi.org/10.2307/3028191

National Council of Teachers of Mathematics. (2014). *Principles to actions: Ensuring mathematical success for all*. Reston, VA: NCTM.

Dr. Joseph Kelly is a graduate of Valdosta State University with a Bachelor of Arts degree in Mathematics, a Master of Science degree in Mathematics Education from Florida State University, a Specialist of Education degree in teacher leadership from Georgia Southwestern State University and a Doctor of Education degree in Secondary Mathematics from Kennesaw State University. Joseph currently teaches AP Calculus at Bainbridge High School. His research interests include mathematical identity, the influence of tracking, and mathematics education in rural communities.

SUPPORTING METACOGNITION IN STEM STUDENTS

TORI WILSON

Abstract

STEM education is rising across the nation as we advance in teaching practices and strategies to enhance the education of learners (Akin-Kostereliogl & Bogzun, 2023). With an upward trend in STEM education, the approach of metacognition is vital as educators prepare the minds of the next generation. In simple terms, metacognition is how a learner thinks about thinking. Throughout my research, I have noticed patterns in how the process of thinking looks differently in a STEM environment. STEM education can act as a catalyst for improving metacognition in adolescents. STEM provides a unique education for students to experience core subject areas cross-curricular to give a holistic approach to learning. With a unique experience, students may think about thinking differently than a student in a traditional classroom. Therefore, I found the importance of researching how to support STEM students by learning how they individually think in a non-traditional environment to extend their capacity for learning.

Supporting Metacognition in STEM Students

Metacognition is the understanding of someone's thought process. Metacognition is matured through various external and internal factors, so teachers can help facilitate growth in each student. I have researched supporting metacognition in STEM (science, technology, engineering, and math) students concentrating on metacognition awareness, self-regulated learning, diversity of strategies, and confidence levels. STEM education takes on a different perspective in contrast to a traditional classroom. For instance, STEM education pursues cross-curricular content in the context of an engineering background. While STEM education is exponentially growing, I found interest in researching how I can support metacognition through the lens of a STEM student. As I have taught math in a STEM program, I have noticed trends in how my students think differently, so their metacognition development can look different. I aim to provide effective support in developing metacognition in my STEM students. Supporting metacognition can help students know how they learn best. Metacognition in STEM is more relevant now than ever because of the increase in this pathway in our country, especially in the state of Georgia. Within my research, my goal is to shed insight into the possibilities of providing students with the necessary resources to improve their metacognition.

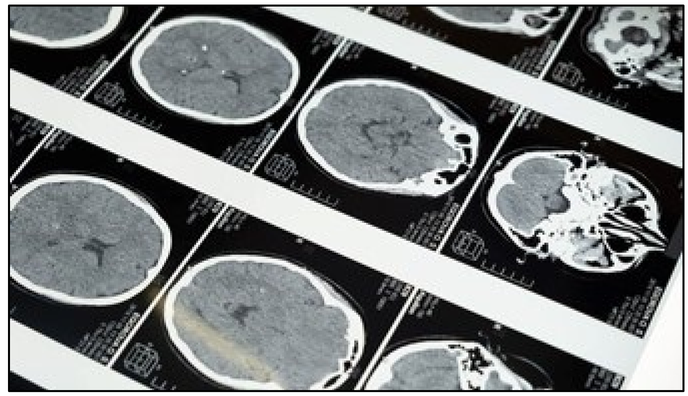
Review of Research

Metacognitive Awareness

Teachers can help students develop their awareness of metacognition to aid in academic and social achievement. There is difficulty in enhancing "the teaching process if we do not have a moderate level of metacognitive awareness about the methods we are using" (Alsmadi et al., 2022). A research study facilitated in Turkey "showed the effect of the metacognitive level of awareness on the regulation of cognition as the metacognitive level of awareness does not stop on clarifying the cognition and increasing the student's self-awareness, it helps with using this awareness and organizing it to enrich the teaching process" (Alsmadi et al., 2022). By equipping students with metacognitive strategies, students can instill definite goals and methods to accomplish their benchmarks (Casanove & Domingo, 2022). In the research of the University of Southampton, they performed a study on 30 independent populations, and their findings "indicated a significantly positive correlation between metacognition and math performance in adolescence" (Cortese at al., 2022). The awareness of metacognition can increase student performance.

Self-Regulated Learning

Once STEM students can comprehend the way they think (metacognition), then they can self-regulate their learning. Self-regulated learning is where a student monitors their tasks and responsibilities (Ainscough, et al., 2021).



The development of metacognition and the "ability to distinguish between effective and ineffective study strategies based on feedback is of utmost importance for secondary school" (Ainscough et al., 2021). A research group defines self-regulation as referring "to learning that is fostered by one's metacognition, strategy adaptability, and motivation" (Ainscough et al., 2021). Self-regulated learning will directly transfer to tertiary schooling and the workforce. Self-regulated learning promotes independent thinking and accomplishing individual goals. Padios and Tobia (2023) harp on "the importance of pairing metacognition with self-efficacy and learning inventory to reach the full potential in a learner's thinking." In a different study, the researchers focus "on the reliability of self-regulated learning in STEM and tie together the concept of self-regulated learning with behavioral observations within the classroom" (Nu'man et al., 2021). Self-regulated learning is derived from metacognition, and self-regulated learning is essential to every student's growth inside and outside of school.

Metacognition Can Unite Diverse Students

Metacognition can unite students with a diversity in cultural barriers and learning strategies. Students can find common ground in the way they think. In the math classroom, students can arrive at solutions in numerous ways. Despite cultural or language barriers, students can unite in how they think about math. A study done on transnational students in Vietnam insists that “students proficient in STEM fields demonstrate strengths such as developing creative and logical thinking, possessing outstanding learning and working capacities, and being able to develop soft skills more” (Briscoe et al., 2021). In addition, “self-regulated learning offers a way to understand individual differences in learning among students, including diverse strategies in delivering modes of learning” (Briscoe et al., 2021). Although metacognition can develop differently in children of diverse backgrounds, children can unite through how they process their thinking (Binning et al., 221).

Confidence Levels

Metacognition in STEM is correlated to learners’ confidence levels and assurance in mathematics and science (Akin-Kostereliogl & Bogzun, 2023). The research at Amasya University shares that “the metacognitive awareness scores were moderately and highly correlated with the self-confidence level” (Akin-Kostereliogl & Bogzun, 2023). Students’ performance is impacted by their confidence. Students who portray confidence are proactive in taking responsibility for their learning (Desender & Sasanguie, 2022). In Desender and Sasanguie’s research, they found that “students with metacognitive awareness can acquire the ability to find and use information, and self-confidence must first develop for the progression of self-efficacy, which is related to metacognition” (2022). Self-confidence is a litmus test for metacognition.

Discussion

Reflective Closing Lessons

The development of metacognition in students can greatly impact their learning, and a true understanding of how they think can enhance their learning (Alsmadi et al., 2022). In my classroom, I have watched students try to learn in a way that is ineffective to them. On the other hand, I have witnessed students who are aware of how they learn best, so they know how to take effective notes and study. Educators can improve metacognitive awareness in their classrooms by implementing reflective closing lessons. The closing lessons should only focus on the effectiveness of the teaching strategy that day. For instance, if I taught a class using a graphic organizer, then I would have my students reflect on how well they learned from that strategy. Then, I would encourage my students to study using the methods most effective for themselves. By building in time for reflection, students are forced to consider how they thought throughout the lesson. This will allow students the freedom of knowing which processes they learn best from.

Choice Boards

Metacognition supports self-regulated learning. Self-regulated learning can enrich learning and build habits to help students in the future (Nu’man et al., 2021). Self-regulated learning puts the focus on the student and allows them to take responsibility for their achievement. Especially at the high school level, I have seen students thrive when they are given freedom in how they learn. For example, teachers can create lessons that allow student choice and voice to be heard and exercised, such as a choice board. A choice board provides flexibility within an assignment's given parameters. Students should learn autonomy in the safety of a classroom, and they should be given the freedom to make mistakes before graduating. Giving students choices in assignments promotes self-regulated learning and metacognition. Allowing students the freedom to choose will enable them to put their reflections into practice. Students can take responsibility for placing themselves in the most effective environment to learn.

Grouping

While there is diversity in metacognition, teachers can use this to their advantage. For example, some students think the same, so teachers can strategically place students in groups dependent on their learning styles. Teachers can choose to place students with similar metacognitions in the same groups. This would allow students to relate with one another due to similar processing methods. On the other hand, teachers can choose to place students with different metacognitions in the same groups. This would allow students to see various approaches to arrive at the same solution (Briscoe et al., 2021). Despite the grouping, metacognition can unite students despite their differences. While students are placed in a group of diverse learners, they can use their strengths to contribute to the group.

Self-Assessment

Metacognition and self-confidence are closely related, and they both impact student performance in the classroom (Akin-Kostereliogl & Bogzun, 2023). In the classroom, I can implement changes simply by being an encouragement to my students. Teachers can make or break students' confidence in their classroom, so I must be careful with my tone and words toward my students. Further, I can have students grade themselves on how well they think they know the information. This process will allow students to see which concepts they are confident in. From there, they can work towards building confidence in other areas to perform their best on assessments. When students can exercise their strengths in metacognition, they can increase their self-confidence in various assignments and tasks.

Conclusion

Overall, supporting metacognition in STEM students can be viewed through the lens of metacognition awareness, self-regulated learning, diversity of students, and confidence levels. Metacognition, thinking about how one thinks, plays a vital role in education. Educators can enhance their students' learning by tending to metacognitive needs. The metacognition developed in adolescents can make an impact throughout adulthood. While metacognition is important for education, the way one thinks is important for a lifetime. Educators have the honor of pouring in students to help them for a lifetime.

References

Ainscough, L., Blackmore, C., Colthorpe, K., Langfield, T., & C., Vitali, J. (2021). A review of self-regulated learning and self-efficacy: The key to tertiary transition in science, technology, engineering and mathematics (STEM). *International Journal of Higher Education, v*(10), 169-177. https://doi.org/10.5430/ijhe.v10n3p169

Akin-Kosterelioglu, M., Bozgun, K. (2023). Self-confidence as the predictor of metacognitive awareness in high school students. *Participatory Education Research, v*(10), 375-388. https://doi.org/10.17275/per.23.20.10.1

Alsmadi, M., Al-Shalabi, E., Haddad, F., Mansour, O., & Tabieh A. A. S. (2022). Metacognitive awareness of STEAM education among primary stage teachers in Jordan. *Journal of Turkish Science Education, v*(19), 1171-1191. https://www.tused.org/10.36681/tused.2022.168

Binning, K.R., Del Toro, J., Qin, X., Wang, M., Zepeda C. D. (2021). More than growth mindset: Individual and interactive links among socioeconomically disadvantaged adolescents' ability mindsets, metacognitive skills, and math engagement. *Child Development, v*(92), 957-976. https://doi.org/10.1111/cdev.13560

Briscoe, K. L., Bush, T., Collins, C., Dang, N. L. T., Tuliao, M., & Yao, C. W. (2021). Exploring STEM undergraduate self-regulated learning at a Vietnamese transnational university. *Journal of Comparative and International Higher Education, v(*13), 6-21. https://www.ojed.org/index.php/jcihe/index

Casanove, V.S., Domingo, M. M. (2022). Metacognitive online reading, navigational strategies, and the reading performance of the grade 11 HUMMS of Pedro T. Mendiola Sr. Memorial National High School. *Journal of Education and Learning, v*(11), 61-73. https://doi.org/10.5539/jel.v11n4p61

Cortese, S., Gosling, C.J., Hadwin, J. A., Highan, P. A., Muncer, G., Wood-Downie, H. (2022). A meta-analysis investigating the association between metacognition and math performance in adolescence. *Educational Psychology Review, v*(34), 301-334. https://doi.org/10.1007/s10648-021-09620-x

Desender, K., Sasanguie, D. (2022). Math anxiety relates positively to metacognitive insight into mathematical decision-making. *Psychological Research, v*(86), 1001-1013.
https://link.springer.com/article/10.1007/s00426-021-01511-8

Nu'man, M., Retnawati, H., Sugiman, Jailani. (2021). Measuring self-regulated Learning in the STEM Framework: A confirmatory factor analysis, *European Journal of Educational Research, v*(10), 2067-2077. https://doi.org/10.12973/eu-jer.10.4.2067

Padios Jr., A. C., Tobia Jr., M. V. (2023). Long distance lab affairs: Physics achievement and metacognition effects of distance laboratories in a senior high school in the Philippines. *Turkish Online Journal of Distance Education, v*(24), 32-46. https://doi.org/10.17718/tojde.1086870

Victoria K Wilson, a teacher at Mill Creek High School, teaches in a STEM program named EPIC (experiential, project-based, innovative, and collaborative/cross-curricular). Throughout her career, she has noticed how students processed material differently and became curious about using differentiation strategies in her instruction to meet their metacognitive needs. She researches metacognition and STEM and plans to implement the strategies she have learned along the way.

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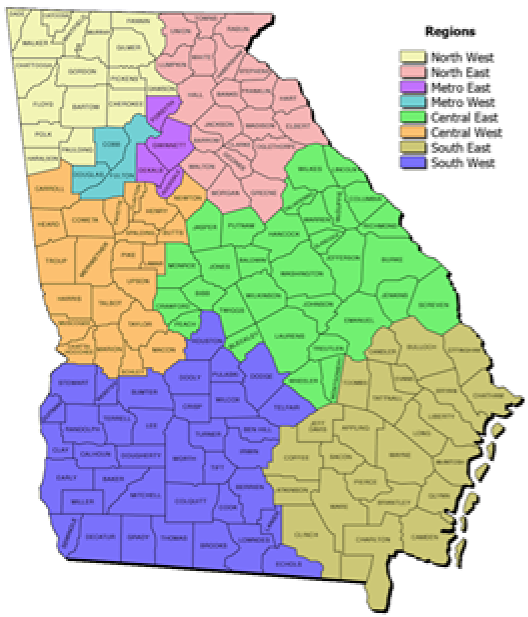

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