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Reflections

Summer 2023 Vol XV, No 1

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A Note from the Editor,

After much anticipation, the first issue of the *peer-reviewed* journal, Reflections, is here. I want to thank all those who submitted their manuscripts for review and approval. In the pages that follow, you will find articles that provide mathematical insights, professional nuggets of wisdom, lessons, and mathematical extensions into STEAM-related fields. What makes this issue so special? For the first time published manuscripts have been approved by a team of reviewers, and these articles are the heart of the issue. Our newsletter highlights organizational updates, but Reflections gives readers a deep dive into mathematics and topics for professional growth. If you would like to become a contributor, please email me, or find out more information at www.gctm.org/reflections/call.

Many thanks,
Dr. Becky Gammill

Editorial Staff

What to become a reviewer? Reach out!

Email the editor at gammillgctm@gmail.com

BACK TO THE BASE-ICS: USING VIRTUAL BASE 10 BLOCKS TO DEVELOP WHOLE NUMBER SUBTRACTION UNDERSTANDING

ABIGAIL LORDEN AND HEIDI EISENREICH

Background

The concept of regrouping when solving subtraction problems can be difficult for some students to grasp, especially when one of the digits in the minuend is zero. However, manipulatives can assist students to make sense of this concept. Manipulatives are useful in many ways and are helpful to students both with and without disabilities (Bouck et al., 2017). Allowing students to interact with hands-on objects is a great way for them to make sense of the concrete stage of the CRA framework (Gersten et al., 2009; Siegler et al., 2010). Although physical, hands-on manipulatives provide many benefits, it can pose some limitations or challenges as students get older (Bouck et al., 2017). Students may be embarrassed to use physical manipulatives, especially if they make a mistake in front of others. Furthermore, families may not have physical base 10 blocks at home. However, many students are familiar with computers, iPads, and other technology, which is important because virtual manipulatives provide accessibility that physical base 10 blocks may not have. Using dynamic manipulatives, whether they are hands-on or virtual, helps students make better connections between the concrete, representational, and abstract stages, as they make sense of difficult concepts.

In this article, which is based on a presentation during the Spring 2022 virtual Georgia Mathematics Conference, we demonstrate how virtual base 10 blocks could be used to support student understanding of whole number subtraction, specifically with regrouping. Due to the online format of the presentation, we focused on supporting teachers to make sense of this concept using virtual manipulatives, specifically base 10 blocks with BrainingCamp. We wanted to share these strategies with Georgia teachers, instructional coaches, paraprofessionals, and administrators.

Although we only utilized the base 10 blocks for this activity, BrainingCamp also has Cuisenaire rods, fraction tiles, geoboards, and many other options that may help you and your students in the classroom or at home. If students are assigned homework, the teacher can create a share code that will already have the manipulative and word problem on the screen when students open it. After solving, the students can share their new code with the teacher, which allows the teacher to look through all the students' strategies when deciding what to include during the class discussion the next day. To get a 30-day free trial and have access to short videos of different tools, see https://www.brainingcamp.com/.

Content Standards

This paper focuses on the following first and second-grade content standards:

- 1.NR.5.1: Use a variety of strategies to solve applicable, mathematical addition and subtraction problems with one- and two-digit whole numbers.
- 2.NR.2.3: Solve problems involving the addition and subtraction of two-digit numbers using part-whole strategies.

These standards focus on solving subtraction problems using a variety of strategies, one of which could be base 10 blocks. The numbers in the examples below can be modified to better fit your students' grade level and current ability by making the numbers larger or smaller. Additionally, this activity connects to Mathematical Practice 7: *Look for and make use of structure* and Mathematical Practice 3*: Construct viable arguments and critique the reasoning of others* because we are using base 10 blocks to make sense of the structure and steps of subtraction, and then explaining and justifying our reasoning.

The task

This activity can be used as an opening task or hook to encourage students to make sense of the problem before solving. You can ask students to explain what is happening in the context of the problem and what resources they may need to solve the problem. To extend the discussion further, small groups can identify possible strategies before they dive into the content and actively solve word problems. The opening task asks "*Victoria had 405 paper clips. She gave 129 paper clips to Sheila. How many paper clips does Victoria have left?"* Some questions that could encourage students to think more deeply about the content are: What is happening in this problem? What do you notice? How would you solve it? Again, remember these numbers could be modified, depending on your students' abilities.

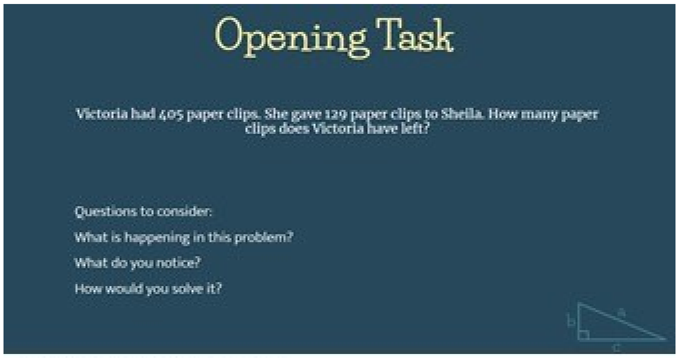


Figure 1: Opening Task

For this opening task, students can engage in a small group discussion by talking about steps they may take to solve this and what the zero might mean when solving the problem. After the discussion, students can begin solving the problem. Teachers can facilitate more discussion in small groups by asking students to explain their thinking process along the way. This is a great time for students to talk about the steps they took, justify why they took those steps, and allow other students in their small groups to ask questions when they do not understand a strategy or do not agree. Once students complete the problem and discuss strategies in their small groups, the teacher can select and sequence strategies to help all students make connections between them. This encourages deeper learning of the content within the whole class discussion.

Figure 2 shows one example of what a student might do. In this particular example, the student skipped the step of showing how they got 10 ones from one hundred (i.e., 1 hundred = 10 tens, and then regrouping one ten for 10 ones). If a student does it this way, this is a good time to explain why it is important to be explicit in your steps and demonstrate each step of the way. Although the individual student may not require this level of explicitness, sharing with the group might cause confusion without all steps.

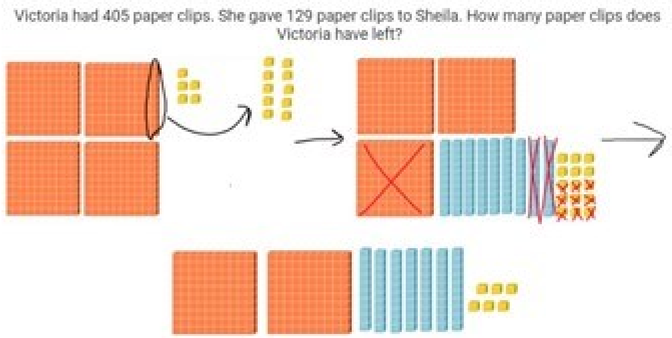


Figure 2: Example of problem-solving process.

Figure 3 shows another example of how a student might solve this problem.

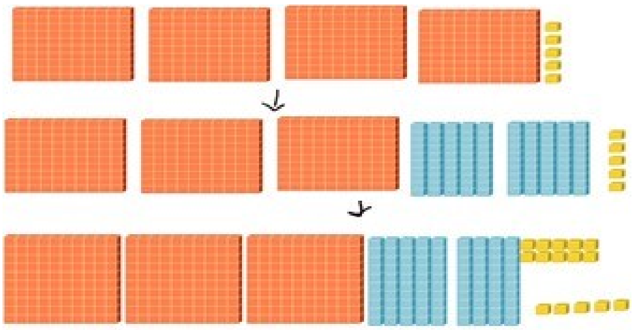
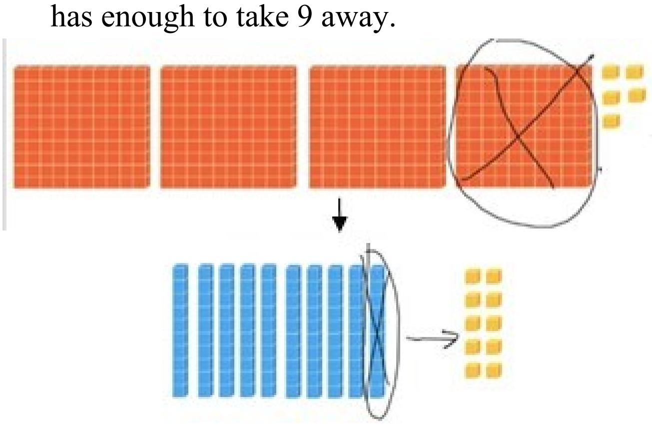


Figure 3.

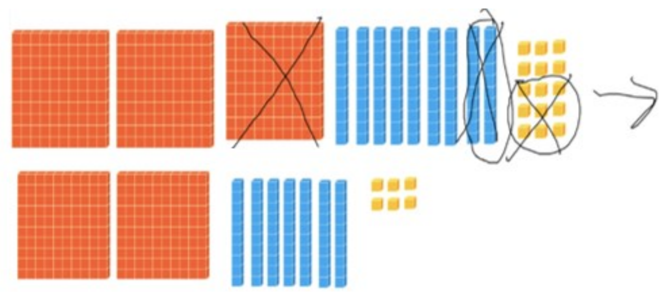
In this example, the student did not show any of their regrouping. Instead, you see some of the base 10 blocks broken up. It is hard to follow along with this example because you don't know what the student is breaking up with and why. You can have this example shown and ask the students what is happening in this problem. Sometimes, whenever students try to understand someone else's work, they realize why they need to be explicit in their strategy.

The steps below are an example of solving this problem using explicit steps.

1. Below, the student showed decomposing 1 hundred into 10 tens and then 1 ten into 10 ones. They do two steps of regrouping, so the student has enough to take 9 away.



2. The student then showed the removal of 129 by crossing out 1 hundred, 2 tens, and 9 ones to show the action of Victoria *giving* 129 paper clips to Sheila. In class discussions, we always want to connect the mathematical steps students used back to the context of the problem. Why are we removing 129? What does it represent? Do we have enough of each base 10 block pieces now to take that amount away?
3. The student then showed their answer with 2 hundreds, 7 tens, and 6 ones. Throughout all these steps, the student made sure to cross off anything they were decomposing or subtracting, which helps to mitigate confusion. Again, we can ask students what this number represents in terms of the context of the problem, which is the 276 paper clips Victoria has left after giving some to Sheila.



In this article, we use whole number language with base ten blocks (thousand, hundred, ten, and one) but when we are talking about the base 10 block pieces we would use large cubes, flat, long, and small cubes to have consistent language (see Figure 4).

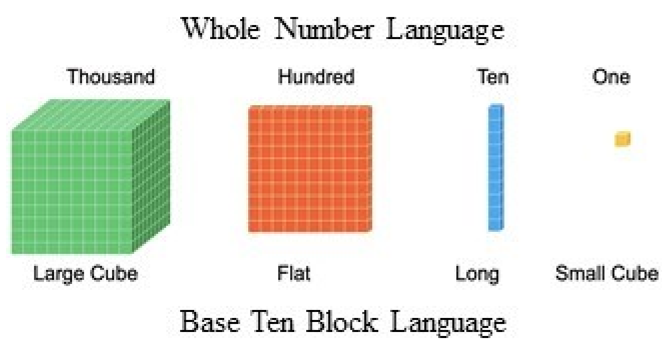


Figure 4.

With whole number language, we would say regroup 1 hundred into 10 tens, and 1 ten into 10 ones. However, we need to make sure students are only using this language when we are working with whole numbers. Once they start working with decimals in later grades, a flat might represent 1 or 10, not 100 anymore. Using both base ten block language (flat, long, small cube) and whole number language (hundred, ten, one) in early grades will help students make the connection when using base 10 blocks to solve decimal problems in later grades. Our base 10 blocks have a 10 to 1 relationship, which works for both whole numbers and decimals.

Conclusion

Regrouping during subtraction can be a difficult skill to master, which is why we should encourage students to use concrete tools and be explicit in their steps when they start drawing representations of those objects. We need to make sure that each step is clear along the way for all students so that there is no room for confusion. By using BrainingCamp or other virtual manipulatives, it can make learning more accessible and productive, not only in the classroom but also at home. Next time you're wondering what to do next in your classroom, consider using BrainingCamp!

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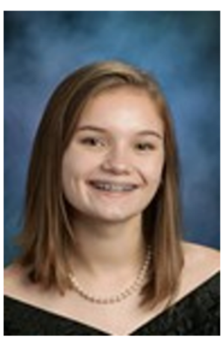

Abigail W. Lorden attends Georgia Southern University as a Special Education major and has an anticipated graduation date of May 2024. She has worked as a research assistant for the past two years. Abigail has presented several times over the last two years at the college, state, and international levels. She plans to focus her future research on using manipulatives and movement to build mathematical understanding in the classroom.

Heidi A. Eisenreich, Ph.D., is an Associate Professor of Mathematics Education at Georgia Southern University in Statesboro, GA. Her research includes helping teachers build a conceptual understanding of mathematics, mentoring preservice teachers in different capacities, and helping parents make sense of strategies their children are learning in elementary mathematics classrooms through parent workshops.

Connecting Art and Mathematics through Symmetry

Ryan Hoffpauir

Introduction

When considering mathematics, most people immediately think of numbers, symbols, and calculations. However, mathematics also relates to numerous other applications such as art. Art especially lends itself to explorations in geometry and can easily be incorporated through several facets of the discipline such as explorations in symmetry.

Before making connections with art, students should be introduced to the four types of symmetry: reflectional, rotational, translational, and glide reflectional (Aichele & Wolfe, 2007). Depending on the grade level of students, symmetry discussions may be limited to just finding lines of symmetry or counting the order of the rotational symmetry. Whereas older students may be able to find angles of rotation or measure distances from a line of symmetry. This relates to the requirements for different levels of standards in Georgia. In the standard MGSE4.G.3 for fourth grade, students need to "Recognize a line of symmetry…Identify line-symmetric figures and draw lines of symmetry" (Georgia Department of Education, 2016, p. 35). Standard MGSE9-12.G.CO.5 elaborates that high school students should be able to "draw the transformed figure (defined by rotation, reflection, or translation) …Specify a sequence of transformations that will carry a given figure onto another" (Georgia Department of Education, 2016, p. 5).

Mandalas

After defining the types of symmetry in level-appropriate ways, students can apply the concepts to several types of artistic expression. One example of such an application is mandalas. Mandalas are designs formed by rotational and reflectional symmetry. This type of art is frequently used in Hindu and Arabic art but also in commercial designs and even in nature.

When using mandalas to explore symmetry, have students first identify what elements of symmetry exist or do not exist in mandalas. This discussion can lead to the understanding that translational symmetry is not possible in this case, but reflectional and rotational symmetry are. Students may then explore several distinct types of mandalas and conclude them. Students should conclude that mandalas will always have rotational symmetry and can sometimes have both reflectional and rotational symmetry. They should also discover that the order of the rotational symmetry (the number of rotations between 0 and 360 degrees that produce the same image) is the same

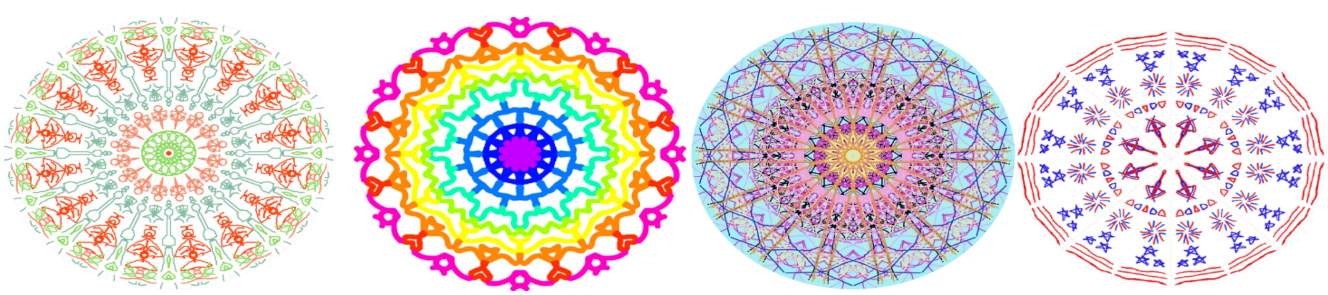

Table 1: Examples of Student created Mandalas

as the order of the reflectional symmetry (the number of lines of reflection).

After students have experience with identifying key features of mandalas, they can create their mandalas. This can be done using various strategies such as making cultural connections or using technology. To create cultural connections, add a requirement to the task that the mandala relates to the student's cultural background in some way. Students can then share mandalas and discuss with their classmates how these elements represent elements of their culture. Technology can be applied by using online software that will copy rotations and reflections for students as they draw (Mandala Maker, 2023). If less digital technology is preferred, polar graph paper can also be used to help students see the sections needed for the mandala while still requiring students to manually create the reflections and rotations for each section.

Tessellations

Another potential art connection to geometry is tessellations. Tessellations are images that can be tiled without having any gaps between them. As with mandalas, several distinct types of tessellations can be created using translations, glide reflections, and rotations as well as by adjusting the base shape. It may be easiest to focus on tessellations with the same base shape such as a quadrilateral. Aichele and Wolfe (2007) describe several types of tessellations based on quadrilaterals and provide an abbreviation system for identifying these types. Translational symmetry is abbreviated with a T and glide reflections are abbreviated with a G. Rotations can be 90-degree rotations around the endpoints of the sides denoted as C_4 or they can be 180-degree rotations around the midpoint of a side which is shortened to C. Each side of the quadrilateral will correspond to one of the transformations. The seven possible tessellations of these types are TTTT, TGTG, GGGG, GCGC, CCCC, TCTC, $C_4C_4C_4C_4$.

Tessellations of these types can be formed by starting with a quadrilateral, the easiest choice being a square. Using sticky notes is an effective method to get a square that is pre-cut to a size that works well for this activity. Students then cut one piece off one side of the paper and then use a transformation to move that piece to a new location and tape it back on the figure. For example, for a translation, a student may cut a piece of the left side of their sticky note and then tape it onto the right side of their sticky note. Translations are the easiest type for students to work with, but students can be challenged to create other types of tessellations using glide reflections, mid-point rotations, and end-point rotations (see Appendix A for a possible assignment and rubric for tessellations).

Like mandalas, tessellations can also be introduced culturally by adding a requirement that the design relates to an element of the student's culture. Many tessellations can be made to look like an object or specific design. Students are first encouraged to make a tessellation using the transformations and then try to make a design out of the result since this is typically easier. However, as students become more familiar with tessellations, they may want to make more challenging designs that relate to their various backgrounds and cultures. This can be a fantastic way for students to express themselves using mathematics (see Appendix B for some student-created examples).

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Mandala Maker (2023). https://mandalamaker.online/

Tessellation Project Resources

The Geometry Palace is looking for ideas for tiling the bathroom floors in a way consistent with the building's rich assortment of interesting geometrical displays. You have been asked to provide some imaginative Escher-style tiles as possibilities for the architect and interior designer to choose from. Your project must include the following parts:

Part 1. Include three different examples representing three different Heesch-type tessellations. Each example should have one page that shows how the tiles fit together with any design elements added. Each example should also include a copy or diagram of the prototile and identify the Heesch type. The prototile is the basic shape that is repeated to form the complete design. Your examples need to be imaginative, creative, and recognizable.

Part 2: Choose one of your examples and describe the process that you used to make the prototile and the design. Include comments on the source of the ideas for the tessellation or a background story for the design.

Part 3: Provide a brief reflection describing how the project relates to the standard below.

Standard 4-Content Knowledge
The teacher understands the central concepts, tools of inquiry, and structures of the discipline(s) he or she teaches and creates learning experiences that make these aspects of the discipline accessible and meaningful for learners to assure mastery of the content.

One way to connect with this standard may be to reflect on how connections with other disciplines such as art can be used to teach mathematical concepts such as symmetry, but other directions can also be taken.

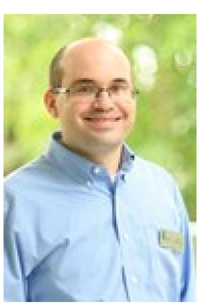
<u>Student Tessellation Examples</u>

Tessellation Student Examples: TTTT Tessellation, TCTC Tessellation, GGGG Tesselation

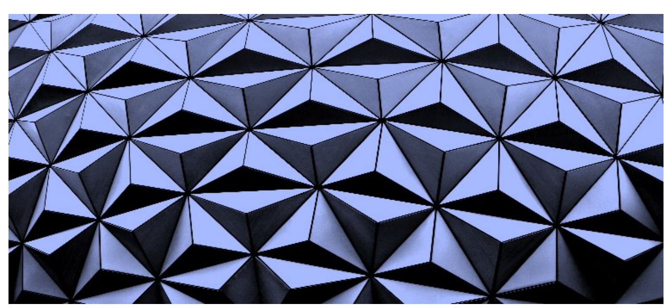
Tessellation Student Examples: $C_4C_4C_4C_4$ Tessellations

Rubric

Category	5 points	3-4 points	1-2 points	0 points
Number of Heesch types	3 types correctly identified	2 types correctly identified	1 type correctly identified	No types correctly identified
Prototiles	3 prototiles provided	2 prototiles provided	1 prototile provided	No prototiles provided
Completed Designs	3 completed designs	2 completed designs	1 completed design	No completed design
Design	All designs are imaginative, creative, and recognizable.	Most designs are imaginative, creative, and recognizable.	Some designs are imaginative, creative, and recognizable.	Designs are not imaginative, creative, or recognizable.
Description	A complete idea source and/or backstory is provided for one example with no significant spelling/grammar errors.	A complete idea source and/or backstory is provided for one example with some significant spelling/grammar mistakes.	An incomplete idea source and/or backstory is provided with some significant spelling/grammar mistakes.	No idea source or backstory is provided for any examples.
Reflection	A reflection is provided that relates to the standard with minimal spelling/grammar errors.	A reflection is provided that relates to the standard but has significant spelling/grammar errors.	A reflection is provided but does not relate to the standard.	No reflection is provided.
SCORE: /25 A: 23-25 points | B: 20-22 points | C: 18-19 points ***Note: C or higher is considered passing*** | D: 15-17 points				



Dr. Ryan Hoffpauir taught several years of high school mathematics and freshmen-level college mathematics in Oklahoma before moving to Georgia in 2020 to teach mathematics courses for elementary education majors at Dalton State College.



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Wednesday	Thursday	Friday
2:00pm • Registration Opens (International Paper)	8:30am Keynote Speaker • Kevin Dykema, NCTM President	8:30am Keynote Speaker • Francis Su, Harvey Mudd College, Author of *Mathematics for Human Flourishing*
3:30- 5:30pm Pre-Conference Sessions • Building Thinking Classrooms • A Taste of Summer Academies • Platinum Sponsor Open Sessions	9:45am- 4:30pm Concurrent Sessions • Speakers Include: Superintendent Richard Woods, Ron Lancaster, Danielle Lanigan, and Members of the Department of Education and the Banneker Society • Exhibit Hall Open	9:45am- 3:30pm Concurrent Sessions • Speakers Include: Darren Starnes, David McMillion, Danielle Lanigan, and members of the Department of Education and the Banneker Society • GAAPMT Conference • Exhibit Hall Open
7:00pm Opening Keynote • Venya Gunya, Former State 4-H President • GCTM Business Meeting	5:00pm • PE at GMC	Travel Safely Home!
TRIVIA at the Pavillion!	7:30pm • Awards, Karaoke, & Dancing at the Pavilion	

Choral Counting in the Elementary Classroom

Jocelyn Robbins

Introduction

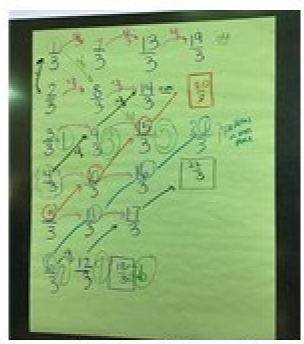
Choral Counting and Counting Collections: Transforming the PreK-5 Math Classroom written by Megan L. Franke, Elham Kazemi, and Angela Chan Turrou is a must-have reference for all elementary teachers of math. Choral Counting and Counting Collections are high-leverage instructional strategies and routines that support numerous foundational mathematical concepts. It may sound like something you might implement in a Kindergarten classroom only, but there are many advantages to using these instructional practices in our upper elementary math classes, too. The focus is on Choral Counting in Kindergarten through Fifth grade.

What is Choral Counting?

Choral Counting is a planned instructional routine where the teacher leads the class in counting out loud together by a certain number to a specific stopping point. The teacher records the count, pausing at predetermined intervals to ask questions like predicting the following number or clearing up miscounting. At the end of the count sequence, the teacher asks the students what patterns they notice in the count. In the primary classroom, choral counts may start at 1 and end at 20. They can begin at 52 and end at 72 or start at 243 and end at 483, skip counting by 10s. In the upper elementary classroom, teachers may skip count by twos, fives, or twenty-fives from a predetermined number, or they may count by fractions, decimals, minutes, hours, or quarts (Turrou et al., 2018).

Why use Choral Counting?

"Counting is fundamental to learning mathematics in elementary school." (Turrou et al., 2018, p. 1). I have heard from many second-grade teachers that many students are challenged with adding and subtracting three-digit numbers. I thought about the First-grade Georgia standards and students' expectations to count to 120. I had an "Aha!" moment. If our students do not count past 120, how can we expect them to be comfortable with three-digit numbers and perform any operations with conceptual understanding? Many of our students have never counted past 120. Choral Counting is an effective instructional practice that will give a high return on a teacher's investment. It is student-centered and engages students in orally counting a sequence and noticing patterns to make sense of the count (Turrou et al., 2018).



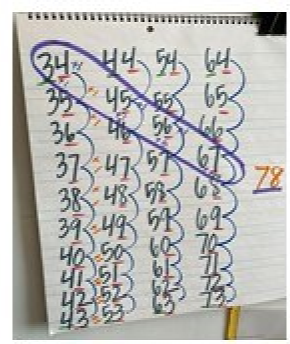


Figure 1: Choral Counting examples.

Upper elementary students always seem to be challenged by fractions and decimals. As educators, we do not ask our students to count by fractions or decimals, but what if we did? Would Choral Counting enhance and support all learners? Yes! Choral Counting provides equity and access for all students in mathematics. If students are not sure of the count, they stay quiet. The record of the count supports students learning English, too. Allowing students to see the count recorded allows for patterns to emerge.

Once a teacher knows how to plan for Choral Counting, determining if you will record the count horizontally or vertically is essential. Then the planning takes about five to ten minutes. There is a free tool that helps plan the count, yet it only works with whole numbers. There are planning tools and graphic organizers available to support teachers, also. To determine if the count has any patterns for students to analyze, the teacher should preplan and try to have equal rows and columns. Writing on chart paper is recommended because the chart is accessible to students. Revisiting these counts can support learning throughout the year by connecting to prior knowledge. Just search Twitter for Choral Counting, and you will find many excellent examples of teachers worldwide using this impactful instructional practice (Turrou et al., 2018).

How do you implement Choral Counting?

Choral Counting is a type of Number Talk! This practice takes anywhere from five to fifteen minutes. All you need is chart paper, a whiteboard, or smart board, and different colored markers. You will need a plan for where you will start the count, how you will count, how many rows and columns you will want, and if you will record horizontally or vertically. Students can sit on the floor on a rug, or older students stay seated. The only caveat is that students use their brains and mouths- no writing for them! Tell the count and give students about thirty seconds to one minute of think time to come up with the first few numbers in the sequence. When you begin counting, record the count (the teacher stays silent and lets the students do the work), pausing to make corrections or to slow the count down or for the students to make predictions. Turn and talks or think-pair-share can be used during this time as well. When the count is completed, ask students to look for patterns or if they notice any patterns in the count. When students share, the teacher uses different color markers for the other students' patterns. Honor all student voices, even if it is a repeated pattern. Always consolidate the learning from the count. It is important to wrap up by reiterating a certain pattern or concept that emerged (Turrou et al., 2018). At first, it may be challenging for students to notice patterns, but over time you will be amazed at how creative students can be! I have had to stop students from finding patterns and promised to return later to their class to hear what else they have noticed.

There are many ways to extend this routine. Once you have completed counting, have the record on the chart, and have patterns that students noticed, the teacher could ask students to journal about one of the patterns and explain it. This way, all students can participate. Invite students to find other patterns or different ones. Primary and Upper elementary students may even need to practice the count with a partner.

I highly recommend the book *Choral Counting and Counting Collections: Transforming the PreK-5 Math Classroom* written by Megan L. Franke, Elham Kazemi, and Angela Chan Turrou for any elementary school teacher of math. It is user-friendly, has practical ideas that are easy to implement, and vignettes of teachers' and students' conversations. I have seen so much growth in just students' mathematical mindsets while implementing this high-impact instructional routine.

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Jocelyn Robbins attended Rutgers University in New Jersey and earned a bachelor's Degree in Psychology, and then a Master's Degree in Early Childhood/Elementary Education. She is currently pursuing an Ed.S in Elementary Education at the University of West Georgia. In 1996, Mrs. Robbins began teaching as an elementary school teacher in New Jersey and later taught in three other states including North Dakota, South Carolina, and Georgia. She currently resides in Fayette County, Georgia where she is the District K-5 Mathematics Instructional Support Teacher who supports 14 elementary schools.

For more information on Choral Counting, please see the resources below.

- Choral Counting (creativemaths.net)
- Choral Counting | TEDD
- Choral Counting – Berkeley Everett
- Illustrative Mathematics
- Choral Counting Recording – GeoGebra
- Choral Counting Tool (stenhouse.com)

WRITING ENGAGING TASKS & A PARAMETRIC FUNCTION ACTIVITY

DAVID HORNBECK

As a math teacher who generally prefers a lecture-centered approach, the last couple of years of engaging in professional development have given me the conviction that no matter how engaging I try to make lectures, I need to do more student-centered lessons. Where do I begin, though? It seems like most activities found online or taught about in professional development sessions are primarily applicable to middle school, Coordinate Algebra, or, at best, Advanced Algebra. Where are those engaging lessons to teach Precalculus, though? How do I help students "discover" the Extended Law of Sines or find beauty and ingenuity in parametric functions?

It is thus that I decided to try to craft my activities – roughly one or two per "unit" of material – to see if I could get students to engage and collaborate *and* become proficient with relatively advanced concepts. The lesson presented here is the first lesson I wrote, as it paved the way for future activities. It makes use of the TI-84 graphing calculator (which we have a class set of) and was inspired by a presentation by Debbie Poss that I attended at the 2022 Georgia Mathematics Conference at Rock Eagle.

The setting of this activity is a precalculus class. It's three weeks before Thanksgiving, and students are fresh off of a unit test on the Law of Sines, the Law of Cosine, and triangle area formulas. With absolutely no prior instruction as to parametric functions, I tell students that they will need a calculator and that they need to find a partner (or group of 3, when necessary). Then, I throw down the activity, and off the students go. Before this activity, students have had plenty of exposure to trigonometric functions, but have not seen conic sections, and their calculator skills are somewhat limited. Therefore, the activity lays out the calculator instructions *very* clearly.

Once the students began working, I was blown away by how accessible the task was. From high-performing students to struggling students, *everyone* was engaged in the task. Because of the brand-new nature of the topic, as well as the calculator skills involved, the task was incredibly democratizing. Nearly all students worked at roughly the same pace, and student-made pairings included both homogeneous and heterogeneous groups. Though such a task could be replicated in Desmos, Geogebra, or any other dynamic software, there seemed to be something about the fact that students had the device in their hands and that computations were not quite as fast as with said software. The writing down of each coordinate based on t seemed to let the concept of a parameter marinate with students. We discussed repeatedly how all of these coordinates seemed to depend on time, as if time was the puppeteer pulling the strings for this function.

Using an ellipse as the first parametric graph was intentional: students organically discussed what made this "oval" shape (a few students knew the term *ellipse*) an oval – why wasn't it circular? This, of course, lays the foundation for ellipses having "two radii," or at least different lengths of major and minor axes. A few students even mentioned that the orbit was similar to that of the Earth around the Sun. Question #10 led to some nice discussions about periodicity and helped students to see how sinusoidal functions can model repetitive behavior, even if it's not circular. Three or four lessons later, we derived the equation of an ellipse *from* its parametric form:

this derivation is far less messy than the geometric definition but is not any less mathematically fruitful. The ball problem is intentional as well, as it got students discussing the differences in the x- and y-subfunctions and what each represented in the context. While my students have always struggled mightily with domain and range, I found that even the students that struggle the most were quickly able to determine both realistic domains. I'm not sure what caused this – maybe it was just the simplicity of the context – but I think it goes back to the idea that having students construct the table one pair of coordinates at a time let the math sink in. Interestingly, students did *not* excel with Practice Question 1(c), though: most students went to the calculator to trace along the curve and heuristically estimate the maximal east distance. Walking around the room and prompting students led to a more excellent discussion about, "Well, could we have done it *without* tracing?" This again led to a review of a previously taught concept: amplitude and range of a sinusoidal function.

By and large, I was impressed with how well the Practice Questions went. Students attempted trial and error with question 2, but they were able to piece together that simply negating both functions wouldn't work; however, some students found that negating *only one* of the functions magically worked, which led to even more discussion as to *why*. Again, we found ourselves able to discuss the symmetric identities for sine and cosine and come to the informal realization that simply negating one function was a mirage: it was that we negated the parameter t.

To be clear, not all of these discussions were able to immediately translate into students magically remembering all of these previously taught concepts; nor were test scores on the eventual unit test off the chart. However, given the difficulty and abstractness of parametric functions, students did far better than I would've expected going into the unit initially. More importantly, this activity demonstrated to me that engaging, meaningful, and, perhaps most importantly, accessible tasks can be created for higher-level math courses, even without loads of, or any, prior instruction. In my steady shift to trying to create a more student-centered classroom, this task was a massive first step.

Activity: Parametric Functions

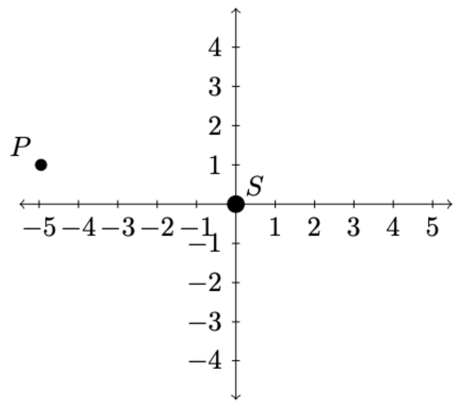
Almost every function you have ever considered has been written in the form $f(x)$, where every point on the graph had coordinates (x, y), where $y = f(x)$. This meant that, at each input x, the function would map x to some output y. Here's a common kind of problem seen in Advanced Algebra:

A ball is thrown from a cliff. The height of the ball is given by $h(t) = -16t^2 + 64t$.
At what time will the ball hit the ground?

This example takes *time* as input and output *height*. When a ball is thrown from a cliff, though, there is also a <u>horizontal</u> distance x from the cliff. This means there are <u>three</u> variables we may be interested in t (time), x (distance from the cliff), and y (height). How can we graph a function with 3 variables?

One way to answer this is through <u>parametric functions</u>. A parametric function $f(t) = (x(t), y(t))$ takes one input – called the <u>parameter</u> – and outputs x and y based on the parameter. This means there are two sub-functions: $x(t)$ for the x-coordinate and $y(t)$ for the y-coordinate.

Activity: Parametric Functions (Continued)



1. Consider the parametric function
$f(t) = (5\cos(t-3), 3\sin(t)+1)$,
which models the orbit of a planet P around a star S for time t in Earth days. Suppose the star S is at the origin $(0,0)$. The units of both x and y are in millions of miles.

2. On your calculator, press **mode**.
First, change to **RADIAN** mode. Then, press down until you reach the **FUNCTION PARAMETRIC POLAR SEQ** ROW.
Select **PARAMETRIC**.

3. Hit **QUIT (2nd MODE)** and press **Y**=. Enter the parametric subfunctions from (1) into **X1T** and Y. Use the **X,T,$\boldsymbol{\theta}$,N** button to type **T**.

4. Hit **2nd window** and change the table setup to start at **TblStart=0** and have a step size a **ΔTbl=0.5**. For **Indpnt,** choose **Ask**.

5. Hit **2nd graph** to access the **Table**. Then, enter **0** for **T**= and hit **enter**. What values do you see for **X1T** and **Y1T**? What do they represent in the graph from (1)?

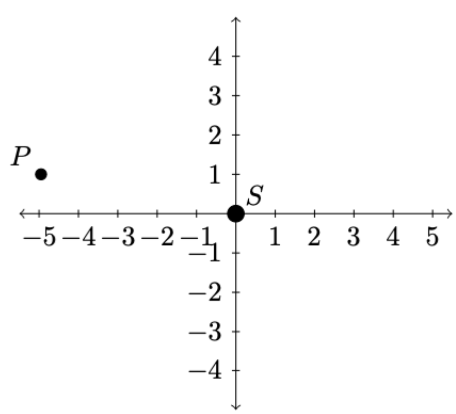
X1T = **Y1T =**

T	X1T	Y1T
0		
1		
2		
3		
4		
5		
6		

6. For each of the values below, find the values of **X1T** and **Y1T** and fill in the table.

7. For each of the x and y values in the table, sketch the points where the planet P will be. Mark each point with its corresponding time value t.

8. Sketch a path through the points you drew. Include arrows that show the direction the planet is moving.



9. Roughly what shape is the path of the planet?

10. Predict the position (x, y) of the planet at each of the times $t = 7$ and $t = 9.5$.

11. Use the **table** to check your answers to (10).

12. Check your answers to (10) again, this time using the graph: hit **2nd trace (calc)** and choose **1:value**.

Parametric functions are ideal for modeling two-dimensional motion based on a parameter t. Let's look at one similar to the Advanced Algebra example, but with 3 variables!

Let $f(t) = (2t - 1, -2t^2 + 32)$ represent the position of a ball being dropped off the top of a building, where x and y are in feet. Let the y-axis represent the vertical plane of the building.

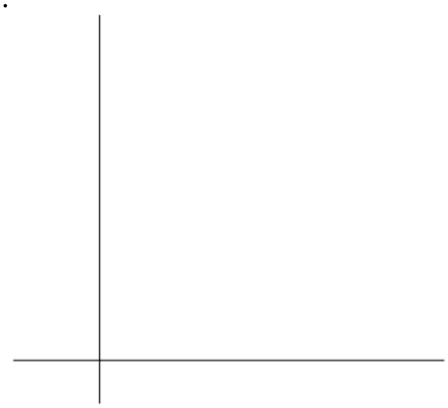
13. Find $x(0)$ and $y(0)$. What do these values mean in context?

14. How high will the ball be after 2 seconds?

15. How far away from the building will the ball hit the ground?

16. Fill out the table of values for $f(t)$ below without a calculator.

t	$x(t)$	$y(t)$
0		
1		
2		
3		
4		
5		
6		
7		



17. Sketch a graph of $f(t)$ on the axes above.

18. After how many seconds will the ball hit the ground?

19. What is the domain of $f(t)$ based on the context? What does this make the *ranges* of $x(t)$ and $y(t)$?

Summary
Parametric functions can model two quantities in terms of a parameter t. This parameter controls both x- and y-coordinates! To draw a parametric function, make a table of values for x and y in terms of t. Then, sketch the points (x, y). Use the table to inform the *direction* the curve is going in, which you can mark with arrows.
$x, y,$ and t will typically mean something in context, so pay attention to what variable each represents.

Practice Questions
1. Researchers on a boat are investigating plankton cells in a sea. The boat is moving on the surface of the sea. At time $t \geq 0$, where t is in hours, the position of the boat is $(x(t), y(t))$, where $x(t) = 662\sin(2t)$ and $y(t) = 880\cos(t)$. Let x be meters east/west and y be meters north/south.

(a) What will the position of the boat be after 2 hours?
(b) Sketch a graph of the boat's path for $0 \leq t \leq 6$.
(c) What is the farthest east ($x > 0$) the boat can get, and what is the first time this will occur?
(d) What is the first time after $t = 0$ that the boat will be back at its starting point?
2.(a) Sketch a graph of the parametric function $f(t) = (\cos(t), \sin(t))$ for $0 \leq t \leq 2\pi$. Include arrows indicating the direction of motion.

(b) Write a new parametric function $g(t)$ that would have the same graph as $f(t)$, but with an *opposite* direction of motion.

Activity: Parametric Functions Solutions & Teacher Notes

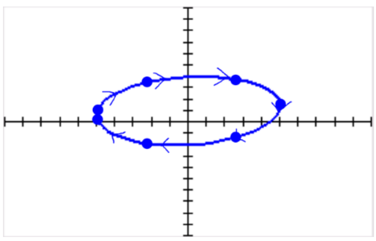
1-4. Generally, students flew through this!

5. $x_{1T} = -4.95, y_{1T} = 1$; the starting position of the planet

6. For each of the values below, find the values of **X1T** and **Y1T** and fill in the table.

T	X1T	Y1T
0	-4.95	1
1	-2.081	3.5244
2	2.7015	3.7279
3	5	1.4234
4	2.7015	-1.27
5	-2.081	-1.877
6	-4.95	0.1618

7.



8-9. Most students said "oval." Walking around the room, you can ask students if they know the formal name.

10. This led to many good discussions. Most students simply chose $(-2.081, 3.5244)$, assuming the period was 6. This is a great time to discuss why the period isn't quite 6, but why it does seem so close!

13. $x(0) = -1, y(0) = 32$. At the time it was dropped, the ball was 1 foot back from the edge of the building and 32 feet off the ground. (Many students thought the -1 couldn't possibly be right, so it's worth reiterating that x here is not time! *Position* can indeed be negative.)

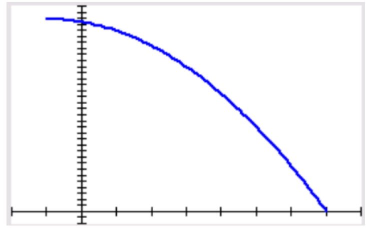
14. $y(2) = 24$ feet

15. $-2t^2 + 32 = 0 \Rightarrow t = \pm 4 \Rightarrow t = 4 \Rightarrow x = 2(4) - 1 = 7$.
Most students used the table for this, but again, students can be pushed to try to determine this using only the functions.

16.

t	$x(t)$	$y(t)$
0	-1	32
1	1	30
2	3	24
3	5	14
4	7	0
5	9	-18
6	11	-40
7	13	-66

17.



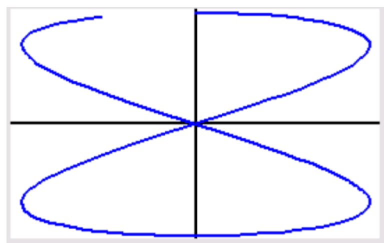
18. $t = 4$

19. The domain is $0 \leq t \leq 4$, and the ranges of $x(t)$ and $y(t)$ are $-1 \leq x(t) \leq 7$ and $0 \leq y(t) \leq 32$. (This was a nice time to discuss difference ways of writing domain and range.)

Practice Questions

1. (a) $(-501, -366.2)$

(b)



(c) $x = 662$ miles east, first occurs at $t = \frac{\pi}{4}$ (Most students approximated this using the trace feature.)

(d) $t = 2\pi$

2. (a) This is the unit circle.

(b)$f(t) = (\cos(-t), \sin(-t)) = (\cos(t), -\sin(t)$

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Assessing the Programming Self-efficacy of Teachers through Professional Development Combining Drones and STEM Activities

Deborah A. McAllister, Jennifer R. Lynberg, and Jared L. Glidden

Abstract

This program focused on beginning work with the Tello EDU drone. The goal was to provide high-quality, teacher professional development to increase knowledge and instructional skills for integrating drones into elementary, middle grades, and secondary classrooms. Activities to be completed during the 4-day, summer workshop were gathered from the Internet and teachers worked in pairs to write an original activity to present to high school students. The program was repeated as a 2-day workshop on consecutive Saturdays. Pre-service and in-service teachers self-evaluated knowledge and skills, before and after the workshop, through a computer programming self-efficacy scale, with questions grouped into five subscales. Participants responded to open-ended questions. Results showed a significant increase in computer programming self-efficacy score and a significant increase in each subscale score. This program was funded through the Tennessee Space Grant Consortium. *The Institutional Review Board of the University of Tennessee at Chattanooga (FWA00004149) has approved this research project # 21-059.*

Introduction

As programming tasks are becoming more commonplace in K-12 classrooms, this program focused on working with pre-service and in-service teachers as they provide experiences to improve the programming and other technical skills of their students. Teachers learned to program and fly the DJI (2023b) Tello EDU drone, which can be programmed to fly automatically using an app or can be flown manually, either by using an app or a physical game controller. The DroneBlocks App (DroneBlocks, n.d.) was used for flying through drag-and-drop, and block-coding, and the DJI (2023a) Tello App was used for flying manually. Each teacher self-evaluated knowledge and skills, before and after a multi-day workshop. The program was presented twice, as a 4-day, summer workshop in the summer of 2022, and as a 2-day workshop on consecutive Saturdays in the winter of 2023.

Review of Literature

Goodnough et al. (2019) collected data regarding teacher pedagogical content knowledge while presenting a unit using drones to study animal habitats. Results showed that self-efficacy levels and understanding of student learning were enhanced through this experience, which, then, translated to increased pedagogical content knowledge and changes in how the teachers planned to conduct their classrooms. Teacher self-efficacy was strengthened as teachers created inquiry-based, classroom environments to engage learners in science.

Walach and Harrell (2021) provided a brief overview of the increasing popularity of drones. They detailed a teacher training workshop to help engineering and technology teachers prepare to use drones in their classrooms. Participants learned to fly in an open space, learned how the drones were built, and modified their drones with more complex electronics. Participants reported a positive experience with the program.

Balogun and Miller (2022) developed, and pilot-tested, a drone club model for out-of-school STEM learning and career pathway exploration for middle and high school students. The activities were primarily related to national security/defense or complex manufacturing. The main goals of the program were to provide students with experience in new technology, inform students about relevant fields to the technology, and provide students with skills that would be attractive to potential employers. K-12 educators and subject-matter experts provided feedback for revision. Feedback topics ranged from safety to instruction to assessment.

Bartholomew et al. (2018) provided reasons why quadcopter drones could be useful for teaching computational thinking, programming, coding, and analytical thinking. They offered a description of one of the activities they conducted, in which drones were used to transport items in a simulation of mountain rescue. They included comprehensive lesson plans, with relevant standards identified.

Talley (2023) reported on an all-girl, middle grades, drone club preparing to compete in a championship. Girls reported interests in flying and programming, and solving problems such as rescuing an animal with a drone. The emphasis was on learning for a future career in a STEM discipline.

Tsai et al. (2019) developed a 16-item, computer programming self-efficacy scale for students above the middle grades level. They reported a reliability alpha of .96 for the self-efficacy scale and a range of .84 to .96 for the subscales. The five subscales included Logical Thinking, Cooperation, Algorithm, Control, and Debug. The subscales were defined on p. 1349-1350 and:

- Logical Thinking "measures students' perceptions of their ability to write a program using logical conditions."
- Cooperation "evaluates students' perceptions of the cooperative nature of a programming task."
- The algorithm "measures students' perceptions of their ability to build up an algorithm for solving a problem independently while programming."
- Control "assesses students' perceptions about their ability to control a program editor."
- Debug "measures students' perceptions of their ability to correct program errors."

Methods

Participants

Pre-service and in-service teachers were recruited for separate, professional development workshops that were held at the University of Tennessee at Chattanooga (UTC) during the summer of 2022 and the winter of 2023. The duration of the summer workshop was 4 consecutive days and included seven participants (two pre-service and five in-service). The duration of the winter workshop was 2 consecutive Saturdays and included nine participants (three pre-service and six in-service). There were no repeat participants from summer to winter.

Program Components

In both workshops, teachers learned to fly a small, quadcopter drone, the Tello EDU (DJI, 2023b), using DroneBlocks (n.d.), a block-coding app, and using Tello (DJI, 2023a), a simulated game controller. A physical game controller, a battery charger, and four extra batteries were provided, as well, for participants to use in the workshop and take to the classroom.

Activities were gathered from several Internet sites (University Corporation for Atmospheric Research - Center for Science Education, 2023; Federal Aviation Administration, 2023; DroneDJ, n.d.; Droneblog, 2023; Tezza et al., 2020; STEM Supplies Blog, n.d.). Sample block coding and/or sample directions for movement were prepared for participant use. Activity topics included height, distance, velocity, direction, flying in a pattern, and payload retrieval.

During the summer workshop, participants developed original activities, which included a hula hoop obstacle course, a blind flying course (find the landing zone using the drone's camera), a blind driving course (around columns), a slalom race course, and coding basics. These activities were presented to high school students who were on campus for the summer of 2022, Governor's School for Prospective Teachers (UTC, n.d.; Tennessee Department of Education, n.d.a) program. The participants in the summer workshop had to learn to use the apps quickly as the high school students joined the workshop on the afternoon of the second day. During the winter workshop, participants practiced programming and/or flying with the sample activities, the hula hoop obstacle course, the blind flying activity, and the slalom racecourse. See Figure 1.

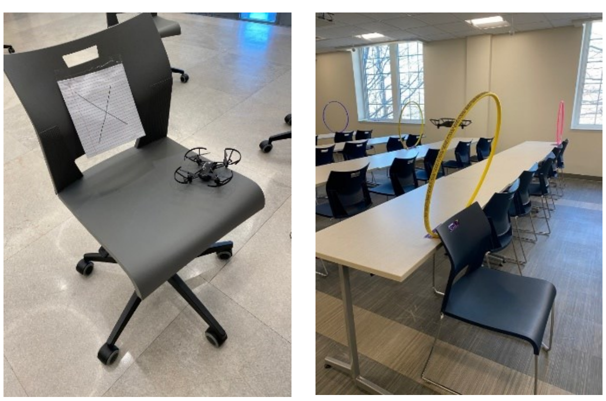

Figure 1: Classroom activities.

Activity topics were correlated to the (a) Tennessee Mathematics Standards (Tennessee Department of Education, n.d.b), which are aligned with the Common Core State Standards (n.d.) for mathematics; (b) the Tennessee Science Standards (Tennessee Department of Education, n.d.c); and (c) the International Society for Technology in Education (n.d.) standards. Concerning mathematics, activities were aligned with topics in the following levels and domains: (a) grades 3-5 measurement and data, (b) grades 5-8 and secondary geometry, (c) grade 8 functions, (d) trigonometric functions, (e) standards for mathematical practice, and (f) literacy standards for mathematics. About science, activities were aligned with cross-cutting concepts and science and engineering practices. Concerning technology, activities were aligned with standards for students, educators, and education leaders. All activities developed by the University Corporation for Atmospheric Research (2023) were correlated to the Next Generation Science Standards; the correlation is available with each activity.

Data Collection

Participants completed the 16-item, computer programming self-efficacy scale, a 6-point, Likert scale (Tsai et al., 2019), at the beginning of the first day and the end of the last day of the workshop. The scale contains five subscales of three or four items considered as a subscale grouping. The five subscales of the instrument include Logical Thinking, Cooperation, Algorithm, Control, and Debug. Participants responded to three, open-ended questions:

1. List 3 things that were learned through this activity.
2. How will you use one or more of these activities in an educational setting?
3. What are some areas for workshop improvement?

The goal was to provide high-quality, teacher professional development to increase knowledge and instructional skills for integrating drones into elementary, middle grades, and secondary classrooms. Measurable objectives included the following:

1. There will be a statistically significant increase in teachers' scores on a 16-item, computer programming self-efficacy survey, between administrations of the instrument.
2. There will be a statistically significant increase in teachers' scores on the five subscales of the computer programming self-efficacy survey, between administrations of the instrument.
3. Responses to open-ended questions will be analyzed for trends.

Results

The results of the Tsai et al. (2019) survey were analyzed separately for the two workshops, as there was a difference in the length of time the participants attended the professional development sessions. With a 6-point scale, the possible range of scores was 6 to 96. For the summer 2022 workshop, the pre-test range was 34 to 90 and the post-test range was 85 to 91. For the winter 2023 workshop, the pre-test range was 30 to 80 and the post-test range was 65 to 95. Results of *t*-tests showed a significant increase in computer programming self-efficacy and significant increases in subscale scores for both workshops. See Figure 2.

Computer Programming Self-Efficacy Results

Workshop	Summer 2022	Winter 2023
Participants	*n*=7	*n*=9
Mean	Pre-test: 58.4 Post-test: 86.9	Pre-test: 60 Post-test: 82
Range	Pre-test: 34-90 Post-test: 85-91	Pre-test: 30-80 Post-test: 65-95
***t*-test**		
Self-efficacy	$p<.01$	$p<.01$
Logical Thinking	$p<.05$	$p<.01$
Cooperation	$p<.01$	$p<.01$
Algorithm	$p<.01$	$p<.01$
Control	$p<.05$	$p<.01$
Debug	$p<.01$	$p<.01$

Figure 2: Self-Efficacy Data Results

Word clouds were generated to display the results of the three, open-ended questions. Results from both workshops were combined. See Figures 3 through 5.

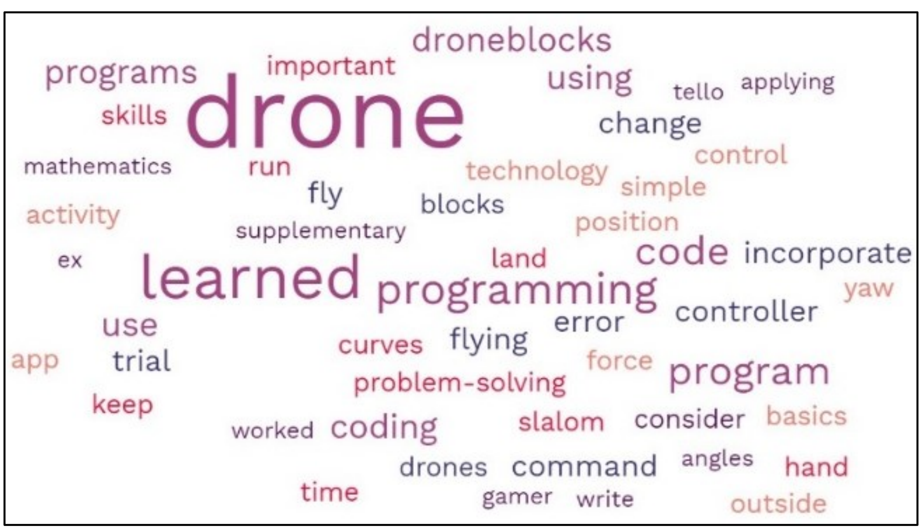

Figure 3: Question 1. List 3 things that were learned through this activity.

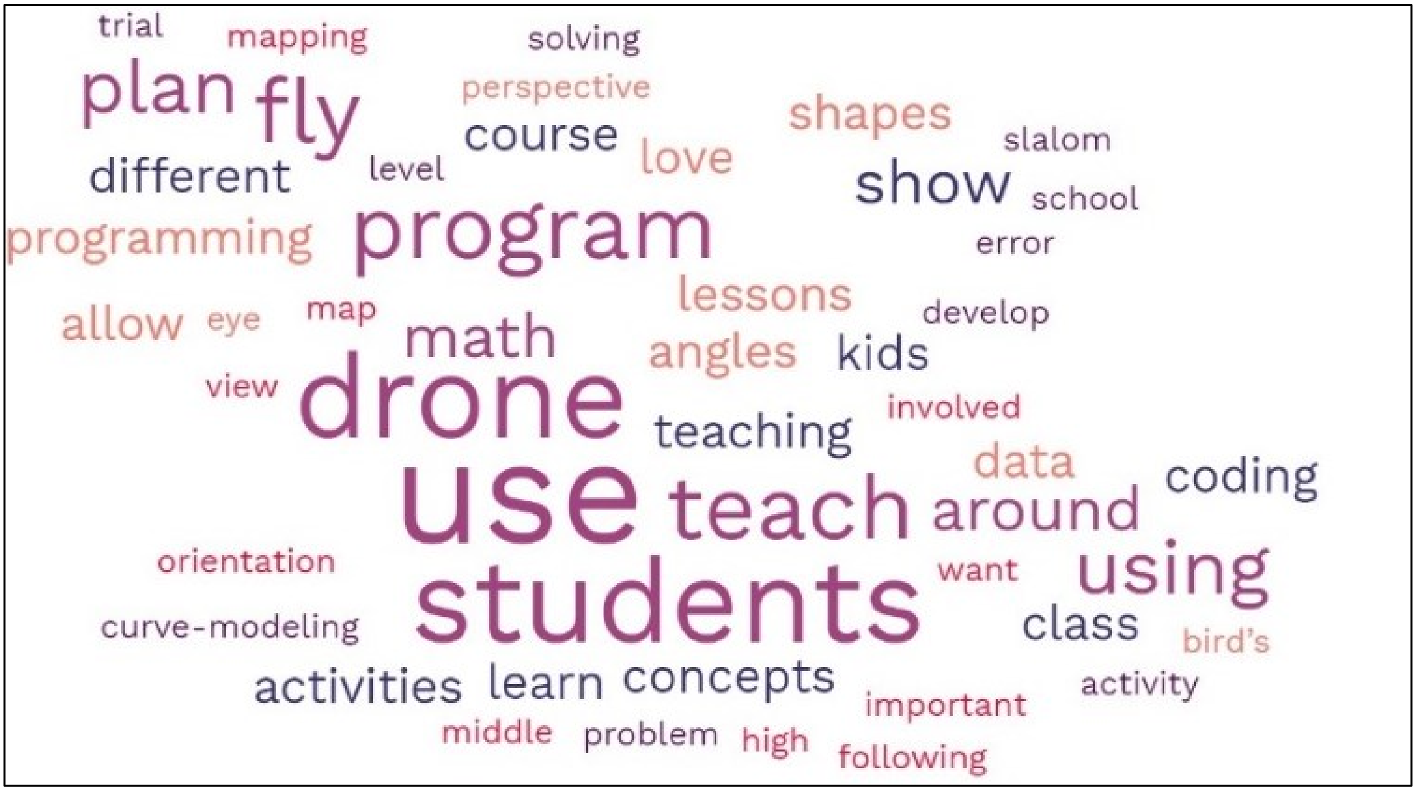

Figure 4: Question 2: How will you use one or more of these activities in an educational setting?

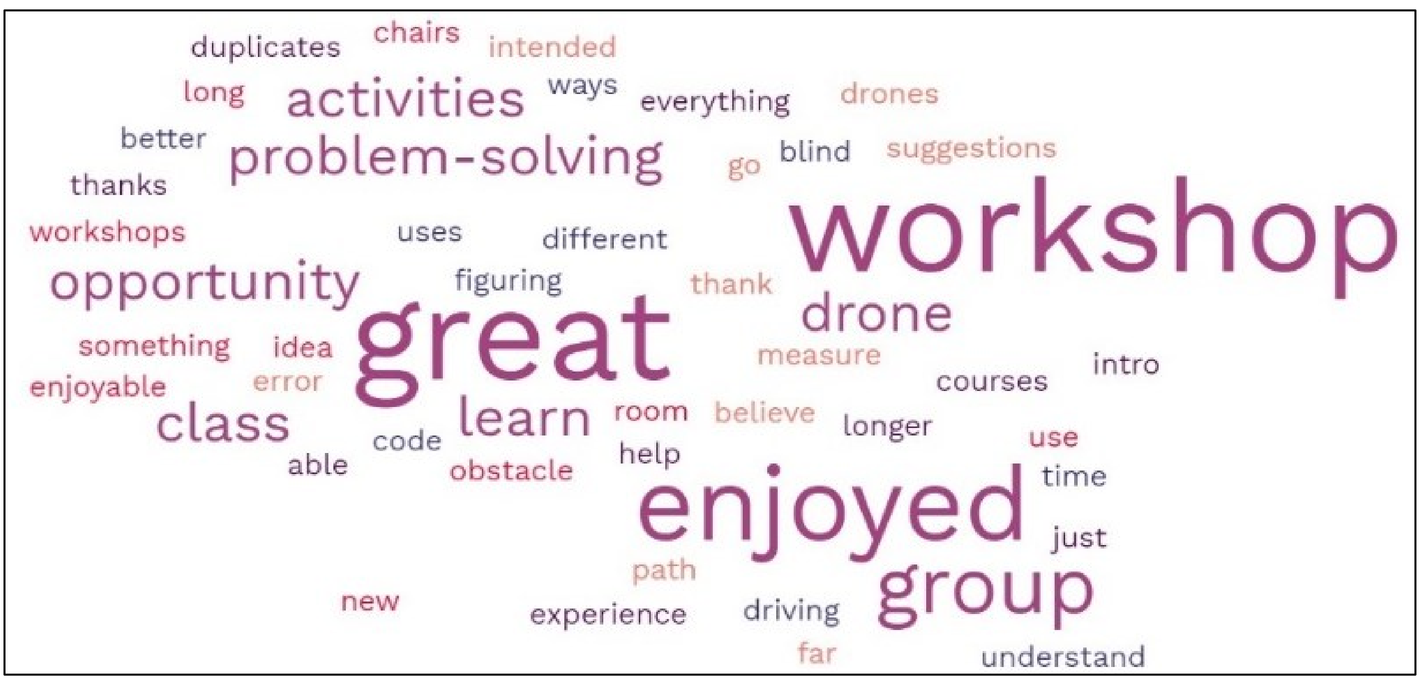

Figure 5 Question 3: What are some areas for workshop improvement?

Discussion

There were significant increases in computer programming self-efficacy, as well as significant increases for each subscale score. In the summer workshop, results were significant at the *p*<.01 level for the self-efficacy scale, as well as the Cooperation, Algorithm, and Debug subscales. The results were significant at the *p*<.05 level for the Logical

Thinking and Control subscales. Six of the seven participants in the summer workshop were teaching mathematics or science or had a degree in a STEM content area. The other participant planned to teach mathematics and had a related first career. About Logical Thinking and Control, there may not have been room for much growth, due to familiarity with the tasks being assessed. The participants worked with logical conditions, daily, while teaching mathematics and science. Some of the participants had a background in computer science or had attended previous robotics workshops and would have had previous knowledge of working with an editor. The use of a drone was new to five of the seven summer participants. In contrast, in the winter workshop, results were significant at the $p<.01$ level for the self-efficacy scale, as well as all of the subscales (Logical Thinking, Cooperation, Algorithm, Control, and Debug). Three of the participants had a mathematics background or taught in the middle grades, and one of the three participants had attended previous robotics workshops. Six of the participants did not have extensive training in STEM content, though one participant had attended previous robotics workshops. This lack of familiarity may have been a reason for more growth to occur in Logical Thinking and Control when compared to the summer participants. The use of a drone was new to all winter participants.

The word cloud for question 1 showed many actions and activities listed. The most prominent ideas showed participants as successful at learning how to program or fly a drone. The word cloud for question 2 showed participants as ready to teach their students how to fly a drone to teach concepts in mathematics. The word cloud for question 3 showed more information related to what was learned and the enjoyment of the workshop. From the original responses, a longer introduction, a measure of error in flying distances, and the elimination of a few duplicate activities on the workshop materials were requested.

Conclusion

Sixteen pre-service and in-service teachers learned to program a drone with a block-coding app and fly a drone using a controller app. Significant increases in computer programming self-efficacy and for the five subscales of Logical Thinking, Cooperation, Algorithm, Control, and Debug were found. Overall, participants were satisfied with the workshop experience and were confident that they would be able to implement the use of drones in elementary, middle grades, or secondary classrooms.

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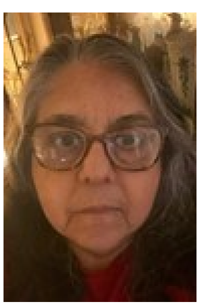

Dr. Deborah A. McAllister is a UC Foundation Professor in the School of Education at the University of Tennessee at Chattanooga, with responsibilities in mathematics education, educational psychology, assessment, and action research. Her research interests include teacher professional development in STEM topics, including drones and robotics. She is a UTC affiliate representative to the Tennessee Space Grant Consortium and is the recipient of the grant which funded this program.

Dr. Jennifer R. Lynberg is an Assistant Profession in the School of Education at the University of Tennessee at Chattanooga, with responsibilities in exceptional education. Dr. Lynberg is the Director of the Governor's School for Prospective Teachers and the UTC Future Ready Institute at Tyner Academy. Her research interests include closing the reading disparity gap for boys of color, creating professional development for teachers surrounding the impact of expressive and receptive language exchanges for students of color, and increasing awareness surrounding the importance of early intervention for students with exceptionalities.

Jared L. Glidden graduated from the University of Tennessee at Chattanooga in 2020 with a B.S. in Psychology. He is currently pursuing his Ed.S. degree in School Psychology at the University of Tennessee at Chattanooga. He will complete the Ed.S. program in May of 2024 before working full-time as a licensed school psychologist. While completing the program, he has served as a graduate assistant under Dr. Deborah McAllister and has been providing support on ongoing research projects.

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Polar Art for Precalculus

Chris Clark

This lesson is intended for students who are in Precalculus. My students complete this lesson after we have introduced polar coordinates and different types of polar graphs. The goal of this lesson is for students to become more acquainted with polar graphs to give them a deeper understanding of them. This is also something that we do at the end of our year in Precalculus and it gives the students some freedom to express themselves through this creative outlet. The assignment takes some time to set up because there are some things that the students need to know about how to craft their piece of art. The instructions for the students are in this file below and give them an idea of how to go about creating their art.

I explain to my students that all of the figures in their artwork needs to be created from a polar graph and needs to include a corresponding equation. They write their equations for figures on the back of their art, and if they change the dimensions in their calculator, they need to explain those changes. They can place these figures anywhere on their canvas. Students also need to include four different types of polar graphs. They can use as many polar graphs as they want. One of the biggest issues that I see all the time is the scaling of their graphs. When they graph their equations on the calculator, they should be scaled the same when they transfer those graphs to their artwork. One last thing that I look for is their presentation of their artwork. This is the subjective part of this assignment as I am looking for students who have put the time in to make their artwork presentable. Students can also include accent lines or shading if they want that do not need to be a polar equation.

In terms of timing for this lesson, I normally take about 20 minutes to explain the artwork and my expectations for them. Then I give them 25 minutes to brainstorm some ideas and to look at different polar graphs. Over the next three class periods (45 minutes in length), I give the students time to work on their artwork. During this time it is important to make sure the students are following the instructions and trouble-shooting any issues that they have. I have found that this is an adequate amount of time for students to complete this assignment.

In the years that I have been doing this, the feedback that I get from students is overwhelmingly positive. This is something that students will remember about Precalculus. I have a wall of art work from previous classes and students are so excited to have their art on my wall. I hope you enjoy this lesson and the examples provided and find time to work this into your Precalculus class when you are talking about Polar graphs!

Figure 1: Polar art student samples.

Polar Art Instructions

1. Put your name on the **back** of your artwork

2. All figures that you draw must represent a polar equation. You may place the figures wherever you want. The figures need to be able to be duplicated on a calculator. If you are using a viewing window other than the standard zoom 6 then you must give your θ min, θ max, x min, x max, etc. so that I can recreate it on my calculator. If you are using Desmos, make sure that all of the equations can be recreated the way they are drawn. (20 pts)

3. You must use at least 4 different types of polar graphs in your artwork – this does not include lines even though you can use lines. (20 pts)

4. List the polar equations on the back of your artwork and explain which figure in your art corresponds to each equation. (20 pts) (I will subtract 2 points for each equation that is missing)

5. The graphs need to be scaled correctly. The sizing of your shapes should be similar on both the art piece and graph. (20 pts)

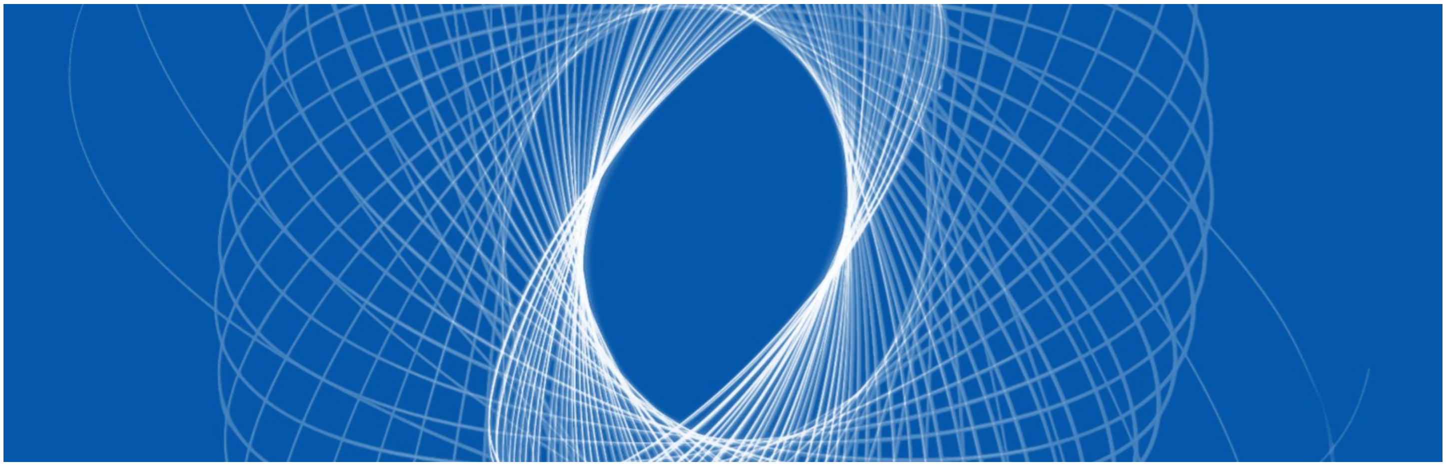
6. There is evidence that you put some thought into your polar art, (I understand that you are not all artistic!). You will be graded on the presentation or your work. (20 pts)

7. You may add accent lines or write words on your artwork

8. All artwork must be original – in other words – don't copy something from the internet.

9. This will be a 100 point test grade – it is due on Friday, May 6th at the beginning of class that day. 5 points will be deducted for each day it is late.

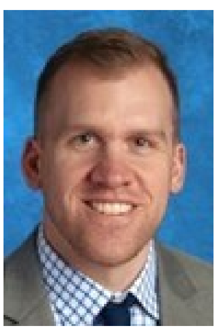
10. Your polar art will be judged and 3 extra points will be awarded to the winner. Depending on the quality of work there might be 2nd and 3rd place awards also. You will be disqualified for extra points if your art is not turned in on time.



Polar Art Rubric

Name ____________________

All figures drawn represent polar equations	_____________	/20
There are four or more types of polar equations	_____________	/20
List Polar Equations for each figure	_____________	/20
All Graphs are Scaled appropriately	_____________	/20
Presentation	_____________	/20
Total	_____________	/100



Chris Clark is in his 13th year of teaching high school mathematics. After completing his undergraduate degree in Finance from Berry College in 2009 and his post-baccalaureate degree from North Georgia College, he started working as a high school math teacher at Providence Christian Academy where he taught Algebra 1, Geometry, Algebra 2, Precalculus and AP Statistics. Chris later completed his M.Ed. from Western Governors University. In 2019, Chris started teaching at Greater Atlanta Christian and currently where teaches Precalculus and AP Statistics and is also pursuing a Specialist Degree in Mathematics Education.

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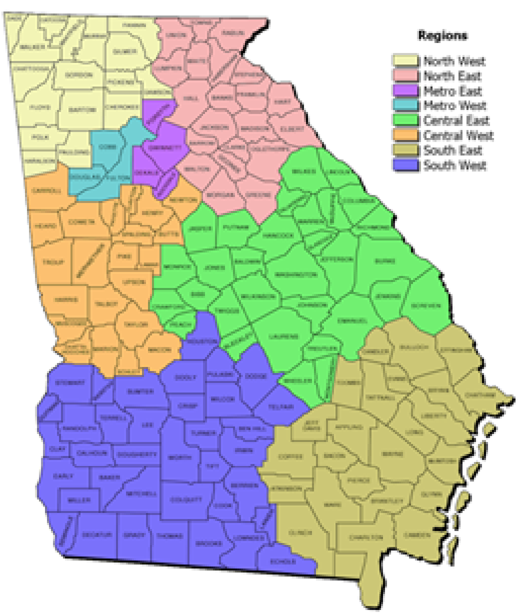

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